MANPOWER STOCKS AND FLOW IN A RANK STRUCTURED-HIRARCHY

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THESIS

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IN A
RANK-STRUCTURED HIRARCHY

by

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Manpower problems of an organization have long been of great concern not only for prediction of the future personnel inventory at different ranks but also for analysing the interaction between states of ranks and indicating chances for promotion. This study is intended to construct and investigate reasonable and tractable models for manpower. That is, models that describe the dynamics of recruitment, advancement, and separation of individuals in a rank-structured system. It is tempting to devise approximate diffusion models for such problems, in order to obtain simple analytical mathematical expressions for

quantities of interest.

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Abstract

Manpower problems of an organization have long been of great concern not only for prediction of the future personnel inventory at different ranks but also for analysing the interaction between states of ranks and indicating chances for promotion. This study is intended to construct and investigate reasonable and tractable models for manpower. That is, models that describe the dynamics of recruitment, advancement, and separation of individuals in a rank-structured system. It is tempting to devise approximate diffusion models for such problems, in order to obtain simple analytical mathematical expressions for quantities of interest.

MAN-POWER STOCKS AND FLOWS

IN A RANK-STRUCTURED HIERARCHY

(Stochastic Models)

TABLE OF CONTENTS

1. Introduction

Manpower problems of an organization have long been of great concern recently, not only for prediction of the future personnel inventory at different ranks but also for analysing the interaction between states of ranks and indicating chances for promotion. Virtually every organization must plan their resources for future demands and uncertainties and manpower is most pertinent among these resources.

Several approaches have been developed for analyzing this kind of problem. Among these, Markovian, or semi-Markovian schemes (1) (2) have been used as well as mathematical expressions for steady state values.

In the real world, the numbers in each rank are correlated rather than independent, and so does between different positions in a rank, e.g. there might be a relatively favored type of individual as well as unfavored ones.

This study is intended to construct and investigate resonable and tractable models for manpower. That is , models that describe the dynamics of recruitment, advancement, and separation(retirement, failure to re-enlist, or discharge) of individuals in a rank-structured system.

It is tempting to devise approximate diffusion models for such problems, in order to obtain simple analytical mathematical expressions for quantities of interest.

In this thesis four stochastic models which describe

the manpower stocks or inventories at each rank are developed. The simplest and most basic model is based on assumed state independence and is presented in section 2A. In the second model, described in section 2B, these two assumption are relaxed. More realistic models that allow for preferencial weighting factors are presented in section 2C and 2D.

For each case, the long term prediction model is developed evolution of the process $\{Q(t), t>0\}$, following the procedure of analytically.

Finally we present the conclusion in chapter 3 along with some recommendations for further work on this topic. Appendices are presented for the algebraic derivations.

2. Models.

A. Model I

In a closed system, recruitment is allowed only for the lowest rank, and advancements are considered for the very next higher rank, but separation can take place from all ranks.

In this model, the organization has a sufficient amount of manpower resources, and the capacity (ceiling) of each rank is unlimited.

The model structure is depicted in Figure 1.

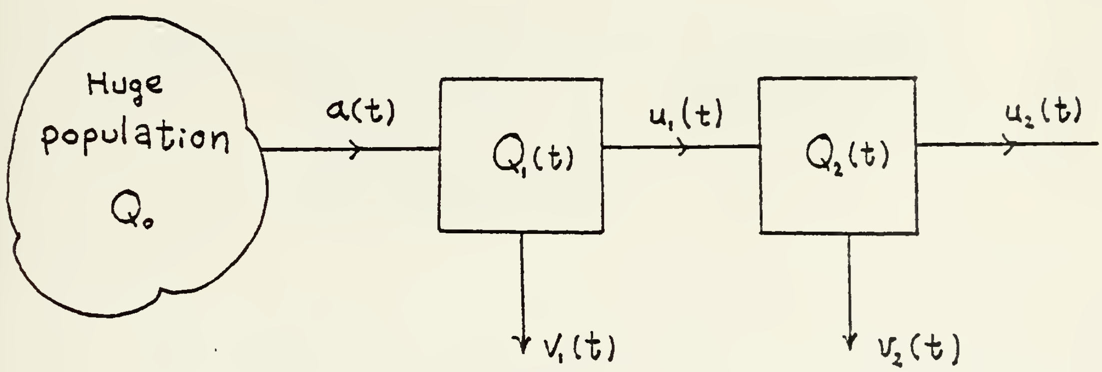


Fig 1.

$a(t)$ is the recruitment rate for the first grade. The personnel who are in the rank-1 ,$Q_1(t)$, can be advanced to the next grade by the rate of $u_1(t)$ or separated from the system by the rate of $v_1(t)$ and so on for the succeeding grade.

The manpower stocks at time t which are in grade i, $Q_i(t)$ are advanced with probability $u_i(t)$, and separated with

probability $v_i(t)$, over the interval $(t,t+dt)$.

We will write down stochastic differential equations of the same form as that given by Gaver and Lehoczky. (3)

Notation

$dQ_i(t)=Q_i(t+dt)-Q_i(t)$ is used.

For grade 1 :

$$
\begin{aligned}
dQ_1(t) &= Q_0 a(t)\,dt - v_1(t)Q_1(t)\,dt - u_1(t)Q_1(t)\,dt \\
&\quad + \sqrt{Q_0 a(t)}\,dW_{\alpha}(t) - \sqrt{Q_1(t)v_1(t)}\,dW_{V_1}(t) \\
&\quad - \sqrt{Q_1(t)u_1(t)}\,dW_{U_1}(t)
\end{aligned}
\qquad \ldots\ldots \quad (1,1)
$$

For grade i, $i=2,3,\ldots,k$

$$
\begin{aligned}
dQ_i(t) &= u_{i-1}(t)Q_{i-1}(t)\,dt - \{u_i(t)+v_i(t)\}Q_i(t)\,dt \\
&\quad + \sqrt{u_{i-1}(t)Q_{i-1}(t)}\,dW_{U_{i-1}}(t) - \sqrt{u_i(t)Q_i(t)}\,dW_{U_i}(t) \\
&\quad - \sqrt{v_i(t)Q_i(t)}\,dW_{V_i}(t)
\end{aligned}
\qquad \ldots\ldots \quad (1,2)
$$

In equations (1,1) and (1,2) the terms $dW_{\alpha}(t)$, $dW_{V_1}(t)$, $dW_{U_1}(t)$, $dW_{V_i}(t)$, and $dW_{U_i}(t)$ are the derivatives of independent standard Wiener precesses, and such as are usually called Gaussian White Noise.

The increment of the population in the 1-st grade is recruitments minus retirements and promotions in $(t, t+dt)$. For small dt the fluctuating (diffusion) component of recruitment is represented by $\sqrt{Q_0 a(t)}\,dW_{\alpha}(t)$, where the scale factor is the standard deviation of a Poisson process with mean $Q_0 a(t)$ and $dW_{\alpha}(t)$ is $N(0,dt)$ and the other noise terms due to promotion and retirement are

obtained from the same rationale.

To study the random noise component, we introduce the standardized noise variables :

$$X_i(t)=\{Q_i(t)-Q_0\, q_i(t)\}/\sqrt{Q_0},\quad i=1,2,\ldots,k$$

$Q_i(t) = \sqrt{Q_0}\, X_i(t) + Q_0\, q_i(t)$ $q_i(t)$ fraction of Q_0 in state i ..... (1,3)

where $Q_0 q_i(t)$ is the mean value of $Q_i(t)$ ≡ stock rank i

The equations (1,1) and (1,2) substituted by (1,3) can be arranged as a system of k ordinary differential equations for the mean value approximation, and a system of n stochastic differential equations for the noise components :

For the mean value approximation,

$q_1(t+1) - q_1(t) =$

$$dq_1(t)/dt=a(t)-\{v_1(t)+u_1(t)\}q_1(t)$$

$$dq_i(t)/dt=u_{i-1}(t)\,q_{i-1}(t)-\{u_i(t)+v_i(t)\}q_i(t)$$

$q_i(t+1) - q_i(t) =$

$i=2,3,\ldots,n$

..... (1,4)

For the noise component ,

$X_1(t+1) - X_1(t) =$

$$dX_1(t)=-\{v_1(t)+u_1(t)\}X_1(t)\,\overset{1}{dt}+\sqrt{a(t)}\,dW_a(t)$$
$$-\sqrt{v_1(t)q_1(t)}\,dW_{v_1}(t)-\sqrt{u_1(t)q_1(t)}\,dW_{u_1}(t)$$

$$dX_i(t)=u_{i-1}(t)X_{i-1}(t)\,dt-\{u_i(t)+v_i(t)\}X_i(t)\,dt$$
$$+\sqrt{u_{i-1}(t)q_{i-1}(t)}\,dW_{u_{i-1}}(t)-\sqrt{u_i(t)q_i(t)}\,dW_{u_i}(t)$$
$$-\sqrt{v_i(t)q_i(t)}\,dW_{v_i}(t)$$

$i = 2,3.$ ..... (1,5)

We express (1,5) as a simple form ;

$$dX_1(t) = -b_1(t)X_1(t)dt + \sqrt{a(t) + b_1(t)q_1(t)}\,dW_1(t)$$

$$dX_i(t) = u_{i-1}(t)X_{i-1}(t)dt - b_i(t)X_i(t)dt$$
$$+\sqrt{u_{i-1}(t)q_{i-1}(t) + b_i(t)q_i(t)}\,dW_i(t)$$

$i=2,3,\ldots$

..... (1,6)

where $b_i(t)=u_i(t)+v_i(t)$ and $dW_i(t)$ is the standard Wiener process associated with grade i, $i=1,2,\ldots$

(Derivation : Appendix A)

We then write noise variables in vector fashion as ;

$$\underline{X}(t)=\{X_1(t),\ X_2(t),\ \ldots,X_n(t)\}'$$

$$\underline{W}(t)=\{W_1(t),\ W_2(t),\ \ldots,W_n(t)\}'$$

$$d\underset{\sim}{X}(t)=\begin{bmatrix} -b_1(t), & 0, & 0 & & & \\ u_1(t), & -b_2(t), & 0 & & & \\ 0, & u_2(t), & -b_3(t) & \cdots & & \\ \vdots & \vdots & \vdots & & & \\ & & \cdots, & 0, & u_{n-1}(t), & b_n(t) \end{bmatrix}\underset{\sim}{X}(t)dt$$

$$+\begin{bmatrix} \sqrt{a(t)+b_1(t)g_1(t)}, & 0 & , & \\ 0 & ,\sqrt{u_1(t)g_1(t)+b_2(t)g_2(t)}, & & \\ & & & \\ & & \cdots & \sqrt{u_{n-1}(t)g_{n-1}(t)+b_n g_n(t)} \end{bmatrix}d\underset{\sim}{W}(t)$$

..... (1,7)

For the mean value of each grade we can solve equation (1,4) grade by the grade or by the computer.
i.e.

$$q_1(t)=\int_0^t \exp\{-\int_s^t (v_1(x)+u_1(x))dx\}a(s)ds$$
$$+q_1(0)\exp\{-\int_0^t (v_1(x)+u_1(x))\}dx$$
$$q_2(t)=\int_0^t \exp\{-\int_s^t (v_2(x)+u_2(x))dx\}\ u_1(s)q_1(s)ds$$
$$+q_2(0)\exp\{-\int_0^t (u_2(x)+v_2(x))dx\}$$
$$\cdots\cdots \quad (1,8)$$

For the noise component we can find $X_1(t)$ from (1,5) i.e.

$$X_1(t)=\int_0^t \exp\{-\int_s^t (v_1(x)+u_1(x))dx\}$$
$$\cdot\sqrt{a(s)+\{v_1(s)+u_1(s)\}q_1(s)}\,dW_1(s)$$
$$+X_1(0)\exp\{-\int_0^t (v_1(x)+u_1(x))dx\}$$
$$\cdots\cdots \quad (1,9)$$

and

$$E\{X_1(t)\}=X_1(0)\exp\{-\int_0^t (v_1(x)+u_1(x))dx\}$$
$$Var\{X_1(t)\}=\int_0^t \exp\{-2\int_s^t (v_1(x)+u_1(x))dx\}$$
$$\cdot\ \{a(s)+(v_1(s)+u_1(s))q_1(s)\}ds$$

For the special case, where, $a(t)=a$, $v_i(t)=v$, and $u_i(t)=u$, after a long period of time $(t\to\infty)$, these values are:

$$E\{X_1(t)\}=0$$

$$Var\{X_1(t)\}=a/(u+v)$$

From (1,8) we can observe the mean number of people in 1-st grade as $t \to \infty$ is $a/(u_1+v_1)$. These results are analogous to an $M/M/\infty$ queueing model in which the mean number of people in the system is $a/(u_1+v_1)$ and the probability that there are n people is Poisson$(a/(u_1+v_1))$. (Ref. (4))

For subsequent grades we have to consider the correlation between $X_i(t)$, $X_j(t)$, $(i \neq j)$. Since we know that these noise terms are sums of Normal random variables whose mean is zero, it follows that $E\{X_i(t)\}=0$ and the variance is equal to the second moment. We describe the moment functions as :

$$m_1(t)=E\{X_1^2(t)\}$$

$$m_2(t)=E\{X_2^2(t)\}$$

$$m_{12}(t)=E\{X_1(t)X_2(t)\} \text{ etc.}$$

and

$$\{m_i(t+dt)-m_i(t)\}/dt = dm_i(t)/dt \text{ as } dt = 0$$
$$i=1,2,\ldots,k$$

Substitute $X_i(t+dt)=X_i(t)+dX_i(t)$ into (1,5), square both sides, and take the expectation, than we can get moment functions.

In this procedure, $E[\{\sqrt{a(t)}\,dW_\alpha(t)\}^2]$ is $a(t)dt$, $Var[dW_\alpha(t)]=E[\{dW_\alpha(t)\}^2]=dt$ since $dW_\alpha(t) \sim N(0, dt)$.

For grade-1 and grade-2,

$$dm_1(t)/dt=a(t)+v_1(t)q_1(t)+u_1(t)q_1(t)$$

$$-2\{v_1(t)+u_1(t)\}m_1(t)$$

$$dm_2(t)/dt=u_1(t)q_1(t)+\{u_2(t)+v_2(t)\}q_2(t)$$

$$-2\{u_2(t)+v_2(t)\}m_2(t)+2u_2(t)m_{12}(t)$$

$$dm_{12}(t)/dt=u_1(t)m_1(t)-\{u_2(t)+v_2(t)+u_1(t)+v_1(t)\}m_{12}(t)$$

$$-u_1(t)q_1(t) \qquad \ldots\ldots \quad (1,10)$$

(Derivation : Appendix B)

The variance and covariance are the solutions of these equations. These solutions can be determined numerically, on computer, and steady state values can be derived explicitly.

NUMERICAL EXAMPLE 1

We present one of the numerical examples;

For, $a=.002$, $u_1=.30$, $v_1=.10$, $u_2=.25$, $v_2=.15$,

and all initial values are zero, then

MODEL-1

TIME	Q-1	Q-2	M-1	M-12	M-2
0.0	0.0	0.0	0.0	0.0	0.0
0.10000E 01	0.16484E-02	0.23082E-03	0.16484E-02	-0.72760E-12	0.23082E-03
0.20000E 01	0.27534E-02	0.71703E-03	0.27534E-02	-0.84886E-11	0.71703E-03
0.30000E 01	0.34940E-02	0.12651E-02	0.34940E-02	-0.70577E-10	0.12651E-02
0.40000E 01	0.39905E-02	0.17815E-02	0.39905E-02	-0.13266E-09	0.17815E-02
0.49999E 01	0.43233E-02	0.22275E-02	0.43233E-02	-0.13266E-09	0.22275E-02
0.59999E 01	0.45464E-02	0.25933E-02	0.45464E-02	-0.13266E-09	0.25933E-02
0.69999E 01	0.46959E-02	0.28834E-02	0.46959E-02	-0.13266E-09	0.28834E-02
0.79999E 01	0.47961E-02	0.31080E-02	0.47961E-02	-0.13266E-09	0.31080E-02
0.89999E 01	0.48633E-02	0.32786E-02	0.48633E-02	-0.34027E-09	0.32786E-02
0.99999E 01	0.49083E-02	0.34065E-02	0.49083E-02	-0.57116E-09	0.34065E-02
0.11000E 02	0.49385E-02	0.35013E-02	0.49385E-02	-0.33833E-09	0.35013E-02
0.12000E 02	0.49588E-02	0.35709E-02	0.49588E-02	-0.28982E-09	0.35709E-02
0.13000E 02	0.49723E-02	0.36217E-02	0.49723E-02	-0.30728E-09	0.36217E-02
0.14000E 02	0.49814E-02	0.36584E-02	0.49814E-02	-0.28982E-09	0.36584E-02
0.15000E 02	0.49875E-02	0.36849E-02	0.49875E-02	-0.28788E-09	0.36849E-02
0.16000E 02	0.49916E-02	0.37038E-02	0.49916E-02	-0.28788E-09	0.37038E-02
0.17000E 02	0.49943E-02	0.37174E-02	0.49943E-02	-0.28594E-09	0.37174E-02
0.17999E 02	0.49962E-02	0.37270E-02	0.49962E-02	-0.28594E-09	0.37270E-02
0.18999E 02	0.49974E-02	0.37338E-02	0.49974E-02	-0.28594E-09	0.37338E-02
0.19999E 02	0.49982E-02	0.37386E-02	0.49982E-02	-0.28982E-09	0.37386E-02

The graphical expressions are available. (graph-1)

THE STEADY STATE MEAN IN EACH RANK OF A MULTI-RANK ORGANIZATION

When an organization maintains a certain policy for recruitment, promotion, and retirement indefinitely, it may have a steady state property (when the rate of flowing into a grade is less than that of flowing out), thus an analytical approximation technique can provide an answer that is quite simple and useful.

For the purpose of investigating such a situation we take $a(t)$, $u_i(t)$, $v_i(t)$ to have constant values in equations (1,4), and solve these under steady-state conditions. This can be done by setting the derivatives equal to zero, and solving.

$$q_1(\infty)=a/b_1$$

$$q_2(\infty)=u_1 a/(b_1 b_2)$$

$$\ldots$$

$$q_n(\infty)=u_1 u_2 \ldots u_{n-1} a/\{b_1 b_2 \ldots b_n\}$$

where $b_i=u_i+v_i$, $i=1,2,\ldots,n$

Special Case

If there is the same chance of promotion and retirement for every grade, u and v, then

$$q_n(\infty)=(a/u)\{u/b\}^n, \quad n=1,2,\ldots$$

and the total number of people in the system is

$$\sum_{i=1}^{n} q_i(\infty)=(a/b)\{[1-(u/b)^n]/[1-(u/b)]\}, \text{ where } (u/b) < 1$$

Let $\bar{q}_n = \sum_{i=1}^{n} q_i(\infty)$,then q is the upper level of total number of people up to rank-n,and the maximum number of people in this organization is

$$\bar{q}_\infty = \sum_{i=1}^{\infty} q_i(\infty) = (a/b)\{1-(u/b)\}^{-1}, \text{ where } (u/b) < 1$$

Sometimes, one is interested in the number of people who are above a certain rank.

$$\bar{q}_\infty - \bar{q}_n = (a/b)\{(u/b)^n/[1-(u/b)]\}, \; n=1,2,\ldots \qquad \ldots\ldots (1,11)$$

This gecmetrically decreasing function represents the relative difficulty in occupying the higher ranks. Moreover, the geometric growth rate (u/b) might be of interest in comparing organizations.

B. Model II

In this model we have improved a ceiling for the population size of each grade, it is not allowed to exceed that level, and promotion rate is proportional to the vacancy of the very next higher grade, where the vacancy is the difference between ceiling and present population.

Let $u_i(t)$ be the rate at which individuals become candidates for promotion from grade i. Some of those in a grade can be promoted, and the rest of them will "give it up", i.e. be forced to leave. We have also a natural loss from each grade, described by the rate $v_i(t)$.
The model structure is depicted in Fig 2.

Q : ceiling of i-th grade

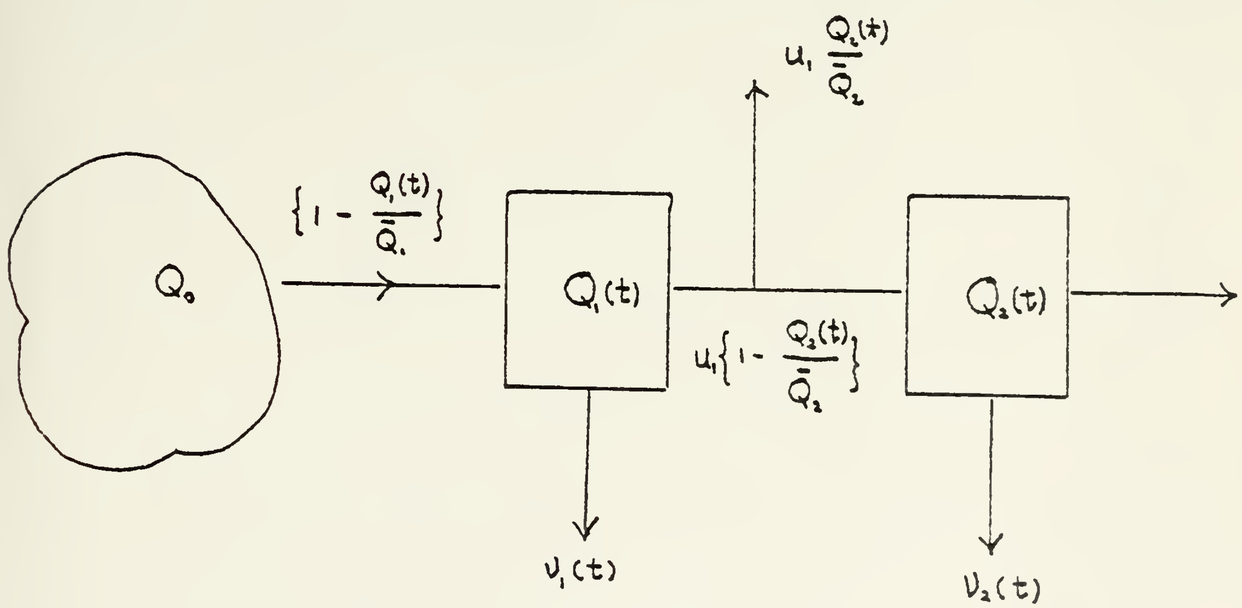


Fig-2

The probability of being recruited from the population is $a(t)\{1-[Q_1(t)/\bar{Q}_1]\}$, the probability of being promoted is modeled by $u(t)\{1-[Q_2(t)/\bar{Q}_2]\}$, and the probability that present grade occupants give up this organization because they failed to advance as candidate is $u_1(t)\{Q_2(t)/\bar{Q}_2\}$.

In order to express the change in a grade we shall first study the change in the mean. The mean increment to the grade is the recruitment or advancement into it, minus those separated and becoming eligible for promotion in $(t, t+dt)$.

Concerning fluctuation (diffusion), we have to consider the terms due to recruitment, promotion, giving up the organization, and natural loss separately since some of these terms are correlated with ones in the next grade. The noise term $dW_{u_1}^{(1)}(t)$ denotes the fluctuation due to losses from the organization, and $dW_{u_1}^{(2)}(t)$ denotes that due to promotion to the next grade.

For the 1-st grade

$$dQ_1(t)=Q_0a(t)\{1-[Q_1(t)/\bar{Q}_1]\}dt-u_1(t)Q_1(t)\{Q_2(t)/\bar{Q}_2\}dt$$
$$-u_1(t)Q_1(t)\{1-[Q_2(t)/\bar{Q}_2]\}dt-v_1(t)Q_1(t)dt$$
$$+\sqrt{Q_0a(t)\{1-[Q_1(t)/\bar{Q}_1]\}}\;dW_a(t)$$
$$-\sqrt{u_1(t)Q_1(t)\{Q_2(t)/\bar{Q}_2\}}\,dW^{(1)}_{u_1}(t)$$
$$-\sqrt{u_1(t)Q_1(t)\{1-[Q_2(t)/\bar{Q}_2]\}}\,dW^{(2)}_{u_1}(t)$$
$$-\sqrt{v_1(t)Q_1(t)}\,dW_{v_1}(t)$$

where $Q_i(t)\leq\bar{Q}_i$, $i=1,2,...$

..... (2,1)

For the 2-nd grade,

$$dQ_2(t)=u_1(t)Q_1(t)\{1-[Q_2(t)/\bar{Q}_2]\}dt$$
$$-u_2(t)Q_2(t)\{Q_3(t)/\bar{Q}_3\}dt$$
$$-u_2(t)Q_2(t)\{1-[Q_3(t)/\bar{Q}_3]\}dt-v_2(t)Q_2(t)dt$$
$$+\sqrt{u_1(t)Q_1(t)\{1-[Q_2(t)/\bar{Q}_2]\}}\;dW^{(2)}_{u_1}(t)$$
$$-\sqrt{u_2(t)Q_2(t)\{Q_3(t)/\bar{Q}_3\}}\,dW^{(1)}_{u_2}(t)$$
$$-\sqrt{u_2(t)Q_2(t)\{1-[Q_3(t)/\bar{Q}_3]\}}\;dW^{(2)}_{u_2}(t)$$
$$-\sqrt{v_2(t)Q_2(t)}\,dW_{v_2}(t)$$

where $Q_2(t)\leq\bar{Q}_2$, $Q_3(t)\leq\bar{Q}_3$

..... (2,2)

We standardize noise variables and Q as k Q to apply the same technique which we had before.
(Appendix A)

We can get two deterministic mean value functions and two stochastic differential equations ;

$$dq_1(t)/dt=a(t)-\{a(t)/k_1+u_1(t)+v_1(t)\}q_1(t)$$

$$dq_2(t)/dt=u_1(t)q_1(t)-\{u_2(t)+v_2(t)\}q_2(t)$$
$$+u_1(t)q_1(t)q_2(t)/k_2$$

..... (2,3)

and

$$dX_1(t)=-\{u_1(t)+v_1(t)+[a(t)/k_1]\}X_1(t)dt$$
$$+\sqrt{a(t)\{1-[q_1(t)/k_1]\}}dW_a(t)$$
$$-\sqrt{q_1(t)q_2(t)u_1(t)/k_2}dW_{u_1}^{(1)}$$
$$-\sqrt{u_1(t)q_1(t)\{1-[q_2(t)/k_2]\}}dW_{u_1}^{(2)}(t)$$
$$-\sqrt{v_1(t)q_1(t)}dW_{v_1}(t)$$

$$dX_2(t)=\{1-[q_2(t)/k_2]\}u_1(t)X_1(t)dt$$
$$-\{u_2(t)+v_2(t)\}X_2(t)dt$$
$$+\sqrt{u_1(t)q_1(t)\{1-[q_2(t)/k_2]\}}dW_{u_1}^{(2)}(t)$$
$$-\sqrt{u_2(t)q_2(t)q_3(t)/k_3}dW_{u_2}^{(1)}(t)$$
$$-\sqrt{u_2(t)q_2(t)\{1-[q_3(t)/k_3]\}}dW_{u_2}^{(2)}(t)$$
$$-\sqrt{v_2(t)q_2(t)}dW_{v_2}(t)$$

..... (2,4)

We express these in a vector fashion

$$\underline{X}(t)=\{X_1(t),\ X_2(t)\}'$$

$$\underline{W}(t)=\{W_a(t),\ W_{u_1}^{(1)}(t),\ W_{u_1}^{(2)}(t),\ W_{u_2}^{(1)}(t),$$

$$W_{u_2}^{(2)}(t),\ W_{v_1}(t),\ W_{v_2}(t)\}'$$

and

$$d\underset{\sim}{X}(t)=\begin{bmatrix} -\left\{\dfrac{a(t)}{k_1}+u_1(t)+v_1(t)\right\} & , & 0 \\ \left\{1-\dfrac{g_2(t)}{k_2}\right\}u_1(t) & , & \left\{v_2(t)+u_2(t)\right\} \end{bmatrix}\underset{\sim}{X}(t)\,dt$$

$$\left[\begin{matrix} \sqrt{a(t)\left\{1-\dfrac{g_1(t)}{k_1}\right\}}, & -\sqrt{\dfrac{u_1(t)}{k_2}g_1(t)g_2(t)}, & -\sqrt{u_1(t)g_1(t)\left\{1-\dfrac{g_2(t)}{k_2}\right\}}, \\ 0, & 0, & \sqrt{u_1(t)g_1(t)\left\{1-\dfrac{g_2(t)}{k_2}\right\}}, \end{matrix}\right.$$

$$\left.\begin{matrix} 0, & 0, & \sqrt{v_1(t)g_1(t)}, & 0 \\ -\sqrt{u_2(t)g_2(t)\dfrac{g_3(t)}{k_3}}, & -\sqrt{u_2(t)g_2(t)\left\{1-\dfrac{g_3(t)}{k_3}\right\}}, & 0, & \sqrt{v_2(t)g_2(t)} \end{matrix}\right] d\underset{\sim}{W}(t)$$

..... (2,5)

Since we are interested in the variance of the noise factor for each grade, we make second moment functions from (2,4) including covariance between $X_1(t)$ and $X_2(t)$

(Appendix B)

$$dm_1(t)/dt = a(t) + \{u_1(t)+v_1(t)\}q_1(t)$$
$$-\{2a(t)q_1(t)/k_1\} - 2\{u_1(t)+v_1(t)\}m_1(t)$$

$$dm_2(t)/dt = \{u_2(t)+v_2(t)+u_1(t)[q_1(t)/k_2]\}\{q_2(t)-2m_2(t)\}$$
$$+u_1(t)q_1(t)\{1-[2q_2(t)/k_2]\}$$
$$+2u_1(t)\{1-[q_2(t)/k_2]\}m_{12}(t)$$

$$dm_{12}(t)/dt = u_1(t)\{1-[q_2(t)/k_2]\}\{m_1(t)-q_1(t)\}$$
$$-\{u_1(t)+u_2(t)+v_1(t)+v_2(t)$$
$$+u_1(t)[q_1(t)/k_2]\}m_{12}(t)$$

$$\cdots\cdots \quad (2,6)$$

We can solve (2,6) together with (2,3) as five first-order differential equations.

NUMERICAL EXAMPLE 2

Let us now consider the example case when;

$a=.001 \quad u_1=.300 \quad v_1=.100 \quad k_1=.002, \quad u_2=.250 \quad v_2=.150 \quad k_2=.0018,$

and all initial values are zero, then

GRAPH-2

TIME	Q-1	Q-2	M-1	M-12	M-2
0.0	0.0	0.0	0.0	0.0	0.0
0.10000E 01	0.69937E-03	0.96009E-04	0.44198E-03	-0.18141E-04	0.88250E-04
0.20000E 01	0.92744E-03	0.24711E-03	0.49737E-03	-0.54923E-04	0.19917E-03
0.30000E 01	0.10364E-02	0.36816E-03	0.49934E-03	-0.77387E-04	0.26787E-03
0.40000E 01	0.10867E-02	0.44913E-03	0.49674E-03	-0.85956E-04	0.30648E-03
0.49999E 01	0.10988E-02	0.49930E-03	0.49512E-03	-0.87947E-04	0.32854E-03
0.59999E 01	0.11061E-02	0.52919E-03	0.49437E-03	-0.87710E-04	0.34143E-03
0.69999E 01	0.11091E-02	0.54659E-03	0.49405E-03	-0.87002E-04	0.34901E-03
0.79999E 01	0.11103E-02	0.55659E-03	0.49392E-03	-0.86374E-04	0.35345E-03
0.89999E 01	0.11108E-02	0.56228E-03	0.49386E-03	-0.85930E-04	0.35604E-03
0.99999E 01	0.11110E-02	0.56549E-03	0.49384E-03	-0.85645E-04	0.35753E-03
0.11000E 02	0.11111E-02	0.56731E-03	0.49383E-03	-0.85471E-04	0.35838E-03
0.12000E 02	0.11111E-02	0.56832E-03	0.49383E-03	-0.85368E-04	0.35886E-03

The graphical expressions are avaiable. (graph-2)

THE STEADY-STATE MEAN IN EACH RANK OF A MULTI RANK ORGANIZATION

For long term prediction, an analytic approximate technique is very attractive. In this case the organization maintains the same policy of recruitment and promotions .

From (2,3)

$$\xi_1(\infty) = \bar{Q}_1 \left\{ \frac{a}{\bar{Q}_1 (u_1 + v_1) + a} \right\}$$

$$\xi_i(\infty) = \bar{Q}_i \left[\frac{\dfrac{\xi_{i-1}(\infty)}{\bar{Q}_{i-1}}}{\dfrac{\bar{Q}_i \, (u_i + v_i)}{\bar{Q}_{i-1} \, u_{i-1}} + \dfrac{\xi_{i-1}(\infty)}{\bar{Q}_{i-1}}} \right]$$

$i = 2, 3, \ldots$

We introduce

$$\tilde{\xi}_i = \frac{\xi_i}{\bar{Q}_i}$$

$$= \frac{\tilde{\xi}_{i-1}}{\rho_i + \tilde{\xi}_{i-1}}$$

where $\rho_i = \dfrac{\bar{Q}_i (u_i + v_i)}{\bar{Q}_{i-1} \, u_{i-1}}$, $\quad i = 2, 3, \ldots$

and let

$$\delta_i = [\ \tilde{q}_i\ ]^{-1}$$

$$= 1 + \rho_i \delta_{i-1}, \qquad i = 2, 3, \ldots$$

..... (2,7)

This formula describes the mean number in rank i as a function of policy and behavioral parameters.

For the same ceiling of each grade and the same chances of promotion and retirement, it is very simple to express the general term of the number of people in a grade.

For $\rho_i = \rho$,

$$\delta_i = \frac{(1-\rho)\rho^{i-1}\delta_1 + (1-\rho^{i-1})}{1-\rho}$$

and

$$q_i(\infty) = \bar{Q} \cdot \frac{(1-\rho)}{1-\rho^{i-1}\{1-\delta_1 - \rho\delta_1\}}$$

Since $\rho > 1$, for the higher grade,

$$q_i(\infty) \doteqdot \bar{Q} \cdot \frac{\rho^2}{1+\delta_1 - \rho\delta_1} \rho^{-i}$$

$$= K\rho^{-i}, \qquad i = 2, 3, \ldots$$

..... (2,8)

where $K = \dfrac{\bar{Q}(1 + \frac{v}{u})}{1 + \frac{v\bar{Q}}{a}}$

This looks like a geometric progression. Thus when $S_i = S$ the distribution of individuals over ranks is geometric in the steady state.

In the following section we develop this model to analyse the interaction between the two kinds of individuals in a grade.

C. Model III (Initial preference Model)

We modify Model II for two kinds of populations, called favored, and unfavored (or normal). In this organization the recruitment rate of the favored population is proportional to the vacancy i.e. number of openings in the grade to which they are candidates for advancement, and for the unfavored group it is proportional to the vacancy, but also to the number of the members of the favored group who are employed therein. It is very interesting to observe one of these examples in an organization which has male and female components ; perhaps the female is favored as a policy choice.

Let G(t), and the subscript g , denotes the favored individuals, and H(t) and the subscript h denotes the unfavored ones.
The model structure is depicted in Figure 3.

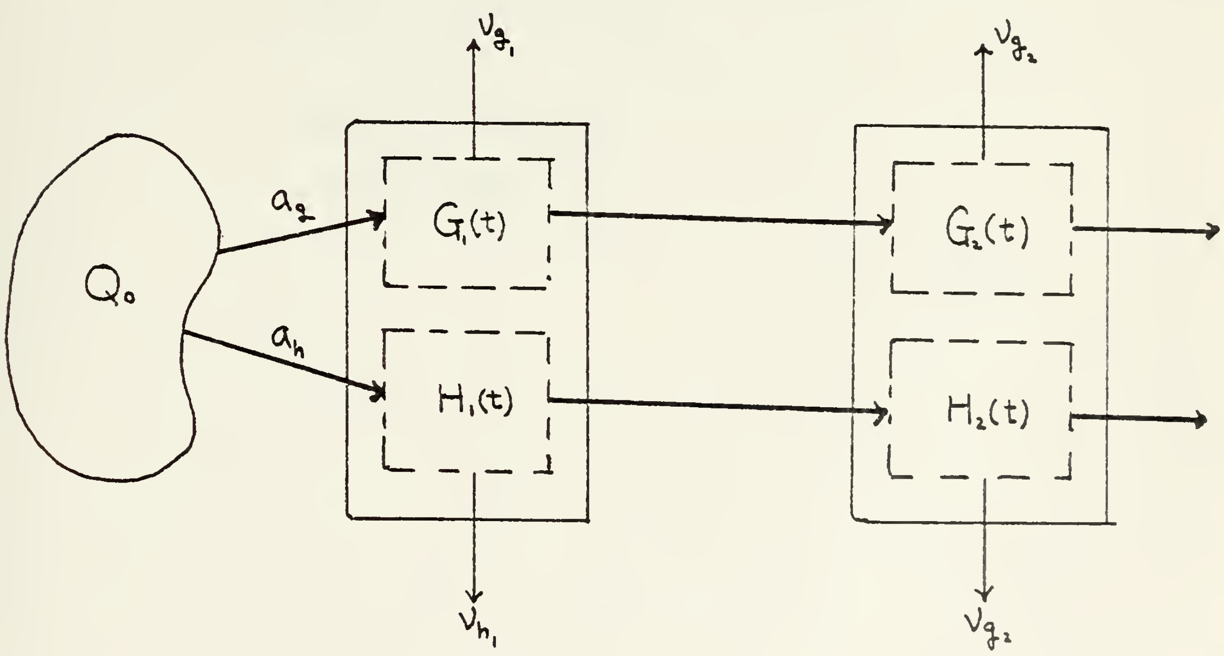


Fig-3

For favored individuals in the 1-st grade,

$$
\begin{aligned}
dG_1(t) = {} & a_g(t)Q_0\{1-[G_1(t)+H_1(t)]/\bar{Q}_1\}dt \\
& -v_{g_1}(t)G_1(t)dt-u_{g_1}(t)G_1(t)dt \\
& +\sqrt{a_g(t)\{1-[G_1(t)+H_1(t)]/\bar{Q}_1\}}\ dW_{a_g}(t) \\
& -\sqrt{v_{g_1}(t)G_1(t)}\,dW_{v_{g_1}}(t)-\sqrt{u_{g_1}(t)G_1(t)}\,dW_{u_{g_1}}(t)
\end{aligned}
$$

where $G_1(t)+H_1(t) \leq \bar{Q}_1$

..... (3,1)

For unfavored individuals in the 1-st grade,

$$
\begin{aligned}
dH_1(t) = {} & a_h(t)Q_0\{G_1(t)/[G_1(t)+H_1(t)]\} \\
& \cdot\{1-[G_1(t)+H_1(t)]/\bar{Q}_1\}dt \\
& -\{v_{h_1}(t)+u_{h_1}(t)\}H_1(t)dt \\
& +\sqrt{a_h(t)Q_0\{G_1(t)/[G_1(t)+H_1(t)]\}\cdot\{1-[G_1(t)+H_1(t)]/\bar{Q}\}}\,dW\ (t) \\
& -\sqrt{v_{h_1}(t)H_1(t)}\,dW_{v_{h_1}}(t) \\
& -\sqrt{u_{h_1}(t)H_1(t)}\,dW_{U_{h_1}}(t)
\end{aligned}
$$

..... (3,2)

To study the random noise component, we introduce the standardized noise variables.

For favored individuals,

$$X_i(t)=\{G_i(t)-Q_0 q_i(t)\}/\sqrt{Q_0}, \qquad i=1,2,\ldots,n$$

For unfavored individuals,

$$Y_i(t)=\{H_i(t)-Q_0 h_i(t)\}/\sqrt{Q_0}, \quad i=1,2,\ldots,n$$

$$\bar{Q}_i=k_i Q_0$$

$$\ldots\ldots \quad (3,3)$$

where $g_i(t)$ and $h_i(t)$ are the mean value of $G_i(t)$ and $H_i(t)$.

From equations (3,1) and (3,2) substituted by (3,3), we can get a system of two ordinary differential equations for the mean value approximations and another system of two stochastic differential equations for noise components;

$$dg_1(t)/dt = a_g(t)\{1-[g_1(t)+h_1(t)]/k_1\}$$
$$-\{v_{g_1}(t)+u_{g_1}(t)\}g_1(t)$$

$$dh_1(t)/dt = \{1/[g_1(t)+h_1(t)]-1/k_1\}a_h(t)g_1(t)$$
$$-\{v_{h_1}(t)+u_{h_1}(t)\}h_1(t)$$

where $g_1(t)+h_1(t) \le k_1$

..... (3,4)

$$dX_1(t) = -\{a_g(t)/k_1\}\{X_1(t)+Y_1(t)\}dt$$
$$-\{v_{g_1}(t)+u_{g_1}(t)\}X_1(t)dt$$
$$+\sqrt{a_g(t)\{1-[g_1(t)+h_1(t)]/k_1\}}\,dW_{a_g}(t)$$
$$-\sqrt{v_{g_1}(t)g_1(t)}\,dW_{v_{g_1}}(t)-\sqrt{u_{g_1}(t)g_1(t)}\,dW_{u_{g_1}}(t)$$

$$dY_1(t) = \{1/[g_1(t)+h_1(t)]-1/k_1\}a_h(t)X_1(t)dt$$
$$-\{v_{h_1}(t)+u_{h_1}(t)\}Y_1(t)dt$$
$$-[a_h(t)g_1(t)/\{g_1(t)+h_1(t)\}^2]\{X_1(t)+Y_1(t)\}dt$$
$$+\sqrt{a_h(t)g_1(t)\{1/[g_1(t)+h_1(t)]-1/k_1\}}\,dW_{a_h}(t)$$
$$-\sqrt{v_{h_1}(t)h_1(t)}\,dW_{v_{h_1}}(t)-\sqrt{u_{h_1}(t)h_1(t)}\,dW_{u_{h_1}}(t)$$

where $g_1(t)+h_1(t) \le k_1$

..... (3,5)

(Derivation : Appendix C)

We then express the noise variables in vector fashion ;

$$\underline{Z}(t)=\{X_1(t),\ Y_1(t)\}'$$

$$\underline{W}(t)=\{W_{a_g}(t),\ W_{a_h}(t),\ W_{u_{g_1}}(t),\ W_{u_{h_1}}(t),\ W_{v_{g_1}}(t),\ W_{v_{h_1}}(t)\}'$$

$$d\underset{\sim}{Z}(t)=\begin{bmatrix} -\left\{\dfrac{a_g(t)}{k_1}+u_{g_1}(t)+v_{g_1}(t)\right\}, & -\dfrac{a_g(t)}{k_1} \\ \left\{\dfrac{1}{g_1(t)+h_1(t)}-\dfrac{1}{k_1}\right\}a_h(t)-\dfrac{a_h(t)\,g_1(t)}{\{g_1(t)+h_1(t)\}^2}, & -\{v_{h_1}(t)+u_{h_1}(t)\}-\dfrac{a_h(t)\,g_1(t)}{\{g_1(t)+h_1(t)\}^2} \end{bmatrix}\underset{\sim}{Z}(t)\,dt$$

$$+\begin{bmatrix} \sqrt{a_g(t)\left\{1-\dfrac{g_1(t)+h_1(t)}{k_1}\right\}}, & 0, & \sqrt{u_{g_1}(t)\,g_1(t)}, & 0, & \sqrt{v_{g_1}(t)\,g_1(t)}, & 0 \\ 0, & \sqrt{a_h(t)\left\{\dfrac{1}{g_1(t)+h_1(t)}-\dfrac{1}{k_1}\right\}g_1(t)}, & 0, & \sqrt{u_{h_1}(t)\,h_1(t)}, & 0, & \sqrt{v_{h_1}(t)\,h_1(t)} \end{bmatrix} d\underset{\sim}{W}(t)$$

Since $X_1(t)$ and $Y_1(t)$ are approximately normal random variables with zero mean, we can find their variances by solving moment function differential equations.

Let

$$m_{X_1}(t)=E[X_1^2(t)]$$

$$m_{X_1Y_1}(t)=E[X_1(t)Y_1(t)]$$

$$m_{Y_1}(t)=E[Y_1^2(t)]$$

and

$$\{X_1^2(t+dt)-X_1^2(t)\}/dt=dm_{X_1}(t)/dt \text{ as } dt\to 0$$

and similarily for $m_{X_1Y_1}(t)$ and $m_{Y_1}(t)$.

By squaring both sides of equation (3,5) and taking expectation, we get a system of three ordinary differential equations of moment function ;

$$dm_{X_1}(t)/dt = a_g(t)\{1-[g_1(t)+h_1(t)]/k\}$$
$$+\{v_{g_1}(t)+u_{g_1}(t)\}g_1(t)$$
$$-2\{v_{g_1}(t)+u_{g_1}(t)+[a_g(t)/k_1]\}m_{X_1}(t)$$
$$-2\{a_g(t)/k_1\}m_{X_1Y_1}(t)$$

$$dm_{Y_1}(t)/dt = a_h(t)g_1(t)\{1/[g_1(t)+h_1(t)]-1/k_1\}$$
$$+\{v_{h_1}(t)+u_{h_1}(t)\}h_1(t)$$
$$+2a_h(t)\{1/[g_1(t)+h_1(t)]-1/k_1\}m_{X_1Y_1}(t)$$
$$-2\{v_{h_1}(t)+u_{h_1}(t)\}m_{Y_1}(t)$$
$$-2a_h(t)g_1(t)/\{g_1(t)+h_1(t)\}^2\{m_{X_1Y_1}(t)+m_{Y_1}(t)\}$$

$$dm_{X_1Y_1}(t)/dt = a_h(t)\{1/[g_1(t)+h_1(t)]-1/k_1\}m_{X_1}(t)$$
$$-a_h(t)[g_1(t)/\{g_1(t)+h_1(t)\}^2]m_{X_1}(t)$$
$$-\{v_{g_1}(t)+v_{h_1}(t)+u_{g_1}(t)+u_{h_1}(t)+[a_g(t)/k\ ]$$
$$+a_h(t)g_1(t)/[g_1(t)+h_1(t)]^2\}m_{X_1Y_1}(t)$$
$$-\{a_g(t)/k_1\}m_{Y_1}(t)$$

$$\ldots\ldots \quad (3,6)$$

In order to get the variance and covariance of the noise component, we can solve a system of three ordinary differential equations instead of two stochastic equations, (3,5), together with (3,4).

For higher grades, up to n-th grade, it is required to set up a system of 2n ordinary differential equations for mean value approximation and another system of $2n+\binom{2n}{2}$ ordinary differential equations derived from a system of 2n stochastic differential equations for noise component.

Taking a six grade military rank-structured organization as an example, we are required to solve 12 equations for mean value approximation and 78 equations for noise component, which makes a system of 90 ordinary simultaneous differential equations.

We will present a simple numerical example for the first grade.

NUMERICAL EXAMPLE 3

Suppose now that we consider the situation when

for favored individuals; $a_g = .001$ $u_{g_1} = .2$ $v_{g_1} = .15$,

for unfavored individuals; $a_h = .002$ $u_{h_1} = .3$ $v_{h_1} = .10$,

and there is a relative-ceiling of .002 assuming non zero initial values, then

MODEL-3

TIME	G-1	H-1	MX-1	M-X1*Y1	MY-1
0.0	0.10000E-02	0.20000E-03	0.20000E-02	0.10000E-02	0.10000E-02
0.10000E 01	0.89934E-03	0.74481E-03	0.75645E-03	-0.38062E-03	0.38693E-03
0.20000E 01	0.78148E-03	0.86351E-03	0.51119E-03	-0.27374E-03	0.35571E-03
0.30000E 01	0.70512E-03	0.91926E-03	0.41260E-03	-0.20184E-03	0.31853E-03
0.40000E 01	0.65915E-03	0.94913E-03	0.36687E-03	-0.16622E-03	0.30226E-03
0.49999E 01	0.63228E-03	0.96541E-03	0.34495E-03	-0.14871E-03	0.29579E-03
0.59999E 01	0.61685E-03	0.97432E-03	0.33397E-03	-0.13984E-03	0.29313E-03
0.69999E 01	0.60810E-03	0.97921E-03	0.32842E-03	-0.13521E-03	0.29197E-03
0.79999E 01	0.60316E-03	0.98190E-03	0.32552E-03	-0.13272E-03	0.29143E-03
0.89999E 01	0.60040E-03	0.98339E-03	0.32396E-03	-0.13137E-03	0.29116E-03
0.99999E 01	0.59886E-03	0.98422E-03	0.32312E-03	-0.13063E-03	0.29102E-03

Graphical expressions are available . (graph-3)

THE STEADY STATE MEAN IN EACH RANK
OF
A MULTI RANK ORGANIZATION*

For the special case, when we have constant parameters for the recruitment, retirement, and promotion, it is attractive to develop an approximation technique for the number of people in each grade.

To observe the steady state value, we set the derivertives equal to zero in equation (3,4), and solve for the ratic of the number of favored population and unfavored ones, grade by grade.

For simplicity we adopt the following notation ;

$$w_i = g_i(\infty)/h_i(\infty)$$

$$b_i = \{u_{g_i} + v_{g_i}\}/\{u_{h_i} + v_{h_i}\}$$

$$a = a_g/a_h\,,\quad u_i = u_{g_i}/u_{h_i} \qquad \ldots\ldots \quad (3,7)$$

By solving (3,4) substituted by (3,7),

$$w_1 = \{a + \sqrt{a^2 + 4b_1 a}\}/(2b_1) \qquad \ldots\ldots \quad (3,8)$$

For the general grades,

$$u_{g_{i-1}} g_{i-1}\{1-(g_i + h_i)/k_i\} - (v_{g_i} + u_{g_i})\, g_i = 0$$

$$u_{h_{i-1}} h_{i-1}\{g_i/(g_i + h_i)\}\{1-(g_i + h_i)/k_i\} - (v_{h_i} + u_{h_i})\, h_i = 0 \qquad \ldots\ldots \quad (3,9)$$

$$u_{i-1}w_{i-1}\{1-(1/w_i)\}=b_i w_i$$

..... (3,10)

and

$$w_i=f_i w_{i-1}+\sqrt{\{f_i w_{i-1}+1\}^2-1} \quad \text{where } f_i=u_{i-1}/(2b_i)$$
$i=2,3,...$

..... (3,11)

It is straight foward to get mean value approximation in steady state, when it exist, to solve (3,4) and (3,9) substituted by (3,8) and (3,11)

$$g_1(\infty)=a_g/\{u_{g_1}+v_{g_1}+(a_g/k_1)(1+1/w_1)\}$$

$$g_i(\infty)=u_{g_{i-1}}g_{i-1}/\{u_{g_i}+v_{g_i}+(u_{g_{i-1}}g_{i-1}/k_i)(1+1/w_i)\},\ i=2,3,...$$

..... (3,12)

and

$$h_i(\infty)=g_i(\infty)/w_i, \quad i=1,2,...$$

..... (3,13)

For the special case, when we have the same policy of promotion and retirement for all rank ;

$$w_i=fw_{i-1}+\sqrt{(fw_{i-1}+1)^2-1} \quad \text{from (3,10)}$$

It is very interesting to observe that the ratio of $g(\infty)$ and $h(\infty)$ has a tendency to converge a certain constant for the very high rank. We can track the value by assuming $w_i=w_{i-1}$, then,

$$uw(1-1/w)=bw$$

$$w=u/(u-b)$$

where $u=u_g/u_h$ $b=(u_g+v_g)/u_h+v_h)$ and $u \neq b$

..... (3,14)

But ,when we have the same policy for the two groups, $f=1/2$,and

$$w=.5w+\sqrt{(.5w+1)^2-1}$$

which diverges.

D. Model IV (Modified Initial Preference Model)

This model is based on the same general concept that was used to develop Model-III. The only difference between Models III and IV is in the preference factor.

We use $G_i(t)/\bar{Q}_i$ instead of $G_i(t)/\{G_i(t)+H_i(t)\}$ as a preference factor, which requires only simple algebra to derive the final result, at least in the steady-state situation. Accordingly, (3,2) is changed to the following :

$$
\begin{aligned}
dH_1(t) = {} & a_h(t)Q_0\{G_1(t)/\bar{Q}_1\}\{1-[G_1(t)+H_1(t)]/\bar{Q}_1\}dt \\
& -\{v_{h_1}(t)+u_{h_1}(t)\}H_1(t)dt \\
& +\sqrt{a_h(t)Q_0\{G_1(t)/\bar{Q}_1\}\{1-[G_1(t)+H_1(t)]/\bar{Q}_1\}}\,dW_{a_h}(t) \\
& -\sqrt{v_{h_1}(t)H_1(t)}\,dW_{v_{h_1}}(t)-\sqrt{u_{h_1}(t)H_1(t)}\,dW_{u_{h_1}}(t) \\
& \text{where } G_1(t)+H_1(t) \le \bar{Q}_1
\end{aligned}
\qquad \ldots\ldots \quad (4,1)
$$

We derive a system of two ordinary differential equations for the mean value approximation and another system of two stochastic differential equations for the noise component form (4,1) together with (3,1).

$$
\begin{aligned}
dg_1(t)/dt = {} & a_g(t)\{1-[g_1(t)+h_1(t)]/k_1\} \\
& -\{v_{g_1}(t)+u_{g_1}(t)\}g_1(t) \\[1ex]
dh_1(t)/dt = {} & -\{v_{h_1}(t)+u_{h_1}(t)\}h_1(t)+a_h(t)g_1(t)/k_1 \\
& -\{a_h(t)/k^2\}\{g_1(t)+h_1(t)\}g_1(t) \\
& \text{where } g_1(t)+h_1(t) \le k
\end{aligned}
\qquad \ldots\ldots \quad (4,2)
$$

and

$$dX_1(t) = -\{a_g(t)/k_1\}\{X_1(t)+Y_1(t)\}dt$$

$$-\{v_{g_1}(t)+u_{g_1}(t)\}X_1(t)dt$$

$$+\sqrt{a_g(t)\{1-[g_1(t)+h_1(t)]/k_1\}}\,dW_{a_g}(t)$$

$$-\sqrt{v_{g_1}(t)g_1(t)}\,dW_{v_{g_1}}(t)$$

$$-\sqrt{u_{g_1}(t)g_1(t)}\,dW_{u_{g_1}}(t)$$

$$dY_1(t) = \{[a_h(t)/k_1]-2g_1(t)-h_1(t)\}X_1(t)dt$$

$$-\{u_{h_1}(t)+v_{h_1}(t)+g_1(t)\}Y_1(t)dt$$

$$+\sqrt{\{a_h(t)/k_1\}\{1-[g_1(t)+h_1(t)]/k_1\}g_1(t)}\,dW_{a_h}(t)$$

$$-\sqrt{v_{h_1}(t)h_1(t)}\,dW_{v_{h_1}}(t)-\sqrt{u_{h_1}(t)h_1(t)}\,dW_{u_{h_1}}(t)$$

$$\cdots\cdots \quad (4,3)$$

We express it as a vector fashion ;

Let

$\underline{Z}(t)=\{X_1(t),\ Y_1(t)\}'$

$\underline{Z}(t)\ \{W_{a_g},\ W_{a_h},\ W_{U_{g_1}},\ W_{V_{g_1}},\ W_{U_{h_1}},\ W_{V_{h_1}},\ \}'$

then

$$d\underset{\sim}{Z}(t)=\begin{bmatrix} -\left\{\dfrac{a_g(t)}{k_1}+U_{g_1}(t)+V_{g_1}(t)\right\}, & \dfrac{a_g(t)}{k_1} \\ \left\{\dfrac{a_h(t)}{k_1}-2g_1(t)-h_1(t)\right\}, & -\left\{U_{h_1}(t)+V_{h_1}(t)+g_1(t)\right\} \end{bmatrix}\underset{\sim}{Z}(t)\,dt$$

$$\begin{bmatrix} \sqrt{a_g(t)\left\{1-\dfrac{g_1(t)+h_1(t)}{k_1}\right\}}, & 0, & \sqrt{U_{g_1}(t)\,g_1(t)}, \\ 0, & \sqrt{\dfrac{a_h(t)}{k_1}\left\{1-\dfrac{g_1(t)+h_1(t)}{k_1}\right\}g_1(t)}, & 0, \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{V_{g_1}(t)\,g_1(t)}, & 0, & 0 \\ 0, & \sqrt{U_{h_1}(t)\,h_1(t)}, & \sqrt{V_{h_1}(t)\,h_1(t)} \end{bmatrix} d\underset{\sim}{W}(t)$$

By the same logic a system of three ordinary differential equations for moments is derived from (4,3).

$$dm_{X_1}(t)/dt=a_g(t)\{1-[g_1(t)+h_1(t)]/k_1\}$$
$$+\{v_{g_1}(t)+u_{g_1}(t)\}g_1(t)$$
$$-2\{v_{g_1}(t)+u_{g_1}(t)+[a_g(t)/k_1]\}m_{X_1}(t)$$
$$-2\{a_g(t)/k_1\}m_{X_1Y_1}(t)$$

$$dm_{X_1Y_1}(t)/dt=\{[a_h(t)/k_1]-2g_1(t)-h_1(t)\}m_{X_1}(t)$$
$$-\{a_g(t)/k_1\}\{m_{Y_1}(t)+m_{X_1Y_1}(t)\}$$
$$-\{v_{g_1}(t)+v_{h_1}(t)+u_{g_1}(t)+u_{h_1}(t)+g_1(t)\}m_{X_1Y_1}(t)$$

$$dm_{Y_1}(t)/dt=\{1-[g_1(t)+h_1(t)]/k_1\}\{a_h(t)/k_1\}g_1(t)$$
$$\{v_{h_1}(t)+u_{h_1}(t)\}h_1(t)$$
$$+2\{[a_h(t)/k_1]-2g_1(t)-h_1(t)\}m_{X_1Y_1}(t)$$
$$-2\{g_1(t)+v_{h_1}(t)+u_{h_1}(t)\}m_{Y_1}(t)$$
$$\ldots\ldots \quad (4,4)$$

We solve (4,4) together with (4,2) to get the mean value approximation and variance and covariance of the noise component.

NUMERICAL EXAMPLE 4

We will now present a simple numerical example for the first grade.

Suppose that for favored inidividuals; $a_g = .001$ $u_{g_1} = .2$ $v_{g_1} = .15$

for unfavored individuals; $a_h = .002$ $u_{h_1} = .3$ $v_{h_1} = .10$,

and the relative-ceiling is .002, assuming non zero initial values, then

MODEL-4A

TIME	G-1	H-1	MX-1	M-X1*Y1	MY-1
0.0	0.16000E-02	0.20000E-03	0.20000E-02	0.30000E-02	0.30000E-02
0.10000E 01	0.92669E-03	0.66259E-03	0.79278E-04	0.85316E-04	0.43297E-02
0.20000E 01	0.81803E-03	0.79333E-03	0.42191E-03	-0.29853E-03	0.18534E-02
0.30000E 01	0.74416E-03	0.84944E-03	0.31927E-03	0.57977E-04	0.12228E-02
0.40000E 01	0.69976E-03	0.87709E-03	0.24637E-03	0.57071E-04	0.11615E-02
0.49999E 01	0.67435E-03	0.89112E-03	0.24205E-03	0.42480E-04	0.10957E-02
0.59999E 01	0.66021E-03	0.89829E-03	0.23942E-03	0.48218E-04	0.10680E-02
0.69999E 01	0.65248E-03	0.90197E-03	0.23708E-03	0.49290E-04	0.10618E-02
0.79999E 01	0.64831E-03	0.90388E-03	0.23651E-03	0.48986E-04	0.10591E-02
0.89999E 01	0.64608E-03	0.90487E-03	0.23634E-03	0.49019E-04	0.10579E-02
0.99999E 01	0.64489E-03	0.90539E-03	0.23624E-03	0.49033E-04	0.10575E-02

The graphical expressions are avaiable. (graph-4)

Assuming zero initial values, then

MODEL-4B

TIME	G-1	H-1	MX-1	M-X1*Y1	MY-1
0.0	0.0	0.0	0.0	0.0	0.0
0.10000E 01	0.58934E-03	0.48672E-03	0.29211E-03	0.26043E-03	0.76807E-03
0.20000E 01	0.69623E-03	0.77818E-03	0.23327E-03	0.10862E-03	0.12258E-02
0.30000E 01	0.69046E-03	0.86109E-03	0.24806E-03	0.35912E-04	0.11531E-02
0.40000E 01	0.67297E-03	0.88719E-03	0.24518E-03	0.46981E-04	0.10890E-02
0.49999E 01	0.66029E-03	0.89719E-03	0.23893E-03	0.49600E-04	0.10716E-02
0.59999E 01	0.65272E-03	0.90161E-03	0.23722E-03	0.48772E-04	0.10633E-02
0.69999E 01	0.64848E-03	0.90374E-03	0.23667E-03	0.48942E-04	0.10594E-02
0.79999E 01	0.64618E-03	0.90481E-03	0.23636E-03	0.49043E-04	0.10581E-02
0.89999E 01	0.64495E-03	0.90536E-03	0.23625E-03	0.49025E-04	0.10576E-02
0.99999E 01	0.64429E-03	0.90565E-03	0.23620E-03	0.49014E-04	0.10574E-02

The graphical expressions are avaiable. (graph-4)

3. CONCLUSION

In this thesis , we have developed four Models to analyze man-power stocks and flows in a rank - structured hierarchy.

We applied the diffusion approximation technique to investigate the fluctuation due to recruitment, retirement, and promoticn rather than Markovian or semi-Markovian schemes.

From the very simplistic Model-I, which has an unlimited ceiling and independent transition policy, we developed Model-II whose recruitment and promotion policy are dependent upcn the present population of the very next grade.

The mean value approximation and variance of the noise component (fluctuation) is the essential factor for planning purpose as well as prediction. It is very useful to analyze the relationship between the fluctuations of two tandem grades as the latter depends upon the promotion policy (rate).

Model-III and Model-IV were developed to represent a more complicated organization than that of Model-II. In this model two individual types (favored and unfavored) compete for the same positions. The relationship between neighboring grades as well as unfavored and favored individuals in the same grade were analyzed.

The approximation technique for the steady state case can be applied to the real world easily because of its simplicity.

Finally, the man-power problem has been concerned with

partial change (modification) in an organization, rather than with the whole structure. In order to analyze these effects, with their fluctuations resulting from a modification of a part of the policies, it is very attractive to develop models using diffusion approximation techniques. They are relatively simple and show promise of representing realistic systems with resonable accuracy.

Appendix-A (Derivation)

Substitute (1,3) into (1,1) and rearrange it for $dX_1(t)$;

$$
\begin{aligned}
dX_1(t) = &-\{v_1(t)+u_1(t)\}X_1(t)\,dt \\
&+\sqrt{Q_0}\,\{a(t)\,dt-[v_1(t)+u_1(t)]q_1(t)\,dt-dq_1(t)\} \\
&-\sqrt{v_1(t)\,\{[X_1(t)/\sqrt{Q_0}]+q_1(t)\}}\,dW_{v_1}(t) \\
&-\sqrt{u_1(t)\,\{[X_1(t)/\sqrt{Q_0}]+q_1(t)\}}\,dW_{u_1}(t) \\
&+\sqrt{a(t)}\,dW_a(t)
\end{aligned}
\qquad \cdots\cdots \quad (A-1)
$$

Since $X_1(t)$ is normal random variable, $dX_1(t)$ is finite , and $\sqrt{Q_0}$ term in (A-1) must be zero as $Q_0 \rightarrow \infty$ (for a huge population), otherwise it will blow up.

It follows that :

$$
dq_1(t)/dt = a(t)-\{v_1(t)+u_1(t)\}q_1(t) \qquad \cdots\cdots \quad (A-2)
$$

and

$$
\begin{aligned}
dX\;(t) = &-\{v_1(t)+u_1(t)\}X_1(t)\,dt \\
&+\sqrt{a(t)}\,dW_a(t)-\sqrt{v_1(t)\,q_1(t)}\,dW_{v_1}(t) \\
&-\sqrt{u_1(t)\,q_1(t)}\,dW_{u_1}(t)
\end{aligned}
\qquad \cdots\cdots \quad (A-3)
$$

For the general grade we rearrange (1,2) substituted by (1,3).

$$
\begin{aligned}
dX_i(t) = {} & u_{i-1}(t)X_{i-1}(t)dt - \{u_i(t)+v_i(t)\}X_i(t)dt \\
& + \sqrt{Q_0}\,\{u_{i-1}(t)q_{i-1}(t)dt - [u_i(t)+v_i(t)]q_i(t) - dq_i(t)\} \\
& + \sqrt{u_{i-1}(t)\{[X_{i-1}(t)/\sqrt{Q_0}]+q_{i-1}(t)\}}\,dW_{U_{i-1}}(t) \\
& - \sqrt{u_i(t)\{[X_i(t)/\sqrt{Q_0}]+q_i(t)\}}\,dW_{U_i}(t) \\
& - \sqrt{v_i(t)\{[X_i(t)/\sqrt{Q_0}]+q_i(t)}\,dW_{V_i}(t)
\end{aligned}
\qquad \ldots\ldots \quad (A-4)
$$

By the same rationale which we had in (A-1) as $Q_0 \to \infty$;

$$
dq_i(t)/dt = u_{i-1}(t)q_{i-1}(t) - \{u_i(t)+v_i(t)\}q_i(t) \qquad \ldots\ldots \quad (A-5)
$$

and

$$
\begin{aligned}
dX_i(t) = {} & u_{i-1}(t)X_{i-1}(t)dt - \{u_i(t)+v_i(t)\}X_i(t)dt \\
& + \sqrt{u_{i-1}(t)q_{i-1}(t)}\,dW_{U_{i-1}}(t) \\
& - \sqrt{u_i(t)q_i(t)}\,dW_{U_i}(t) \\
& - \sqrt{v_i(t)q_i(t)}\,dW_{V_i}(t) \\
& i=2,3,\ldots
\end{aligned}
\qquad \ldots\ldots \quad (A-6)
$$

Since noise terms in a grade are independent of each other, and the standard Wiener process $dW(t)$ is $N(0,dt)$ and its scale factor is the standard deviation of a Poisson process, we can express (A-3) and (A-6) in a simple form, i.e.

$$\sqrt{d_a^2}\,dW_a(t)+\sqrt{d_b^2}\,dW_b(t)=\sqrt{d_a^2+d_b^2}\,dW(t)$$

therefore

$$dX_1(t)=-b_1(t)X_1(t)dt+\sqrt{a(t)+b_1(t)q_1(t)}\,dW_1(t)$$

$$dX_i(t)=u_{i-1}(t)X_{i-1}(t)dt-b_i(t)X_i(t)dt$$
$$+\sqrt{u_{i-1}(t)q_{i-1}(t)+b_i(t)q_i(t)}\,dW_i(t)$$
$$i=2,3,...$$

where

$$b_i(t)=v_i(t)+u_i(t)$$
$$i=1,2,...$$

$$..... \quad \text{(A-7)}$$

The subscript for the Wiener process represents its grade.

Appendix B

(Derivation of Moment Function)

To solve the variance and covariance of the noise component, we derive a system of simultaneous ordinary differential equation for moments from the stochastic differential equations (1,6).

$$X_i(t+dt)=X_i(t)+dX_i(t) \qquad \ldots\ldots \quad (B-1)$$

Substitite (B-1) into (1,6) and square both sides and take the expectation.

$$m_1(t)=E[X_1^2(t)]$$

$$m_2(t)=E[X_2^2(t)]$$

$$m_{12}(t)=E[X_1(t)X_2(t)]$$

and

$$\{E[X_1^2(t+dt)]-E[X_1^2(t)]\}/dt=dm_1(t)/dt$$

as dt approaches to zero.

For the 1-st grade ;

$$X_1(t+dt)=X_1(t)-b_1(t)X_1(t)dt + \sqrt{a(t)+b_1(t)q_1(t)}\,dW_1(t) \qquad \cdots\cdots \quad (B-2)$$

and

$$X_1^2(t+dt)=X_1^2(t)+b_1^2(t)X_1^2(t)(dt)^2$$
$$+\{a(t)+b_1(t)q_1(t)\}dt-2b_1(t)X_1^2(t)dt$$
$$+2\sqrt{a(t)+b_1(t)q_1(t)X_1(t)}\,dW_1(t)$$
$$-2b_1(t)\sqrt{a(t)+b_1(t)q_1(t)X_1(t)}\,dW_1(t)$$

and

$$\{E[X_1^2(t+dt)]-E[X_1^2(t)]\}/dt :$$
$$b_1^2(t)E[X_1^2(t)]dt$$
$$+\{a(t)+b_1(t)q_1(t)\}-2b_1(t)E[X_1^2(t)]$$
$$+2\sqrt{a(t)+b_1(t)q_1(t)}\,E[X_1(t)dW_1(t)]/dt$$
$$-2b_1(t)\sqrt{a(t)+b_1(t)q_1(t)}\,E[X_1(t)dW_1(t)]/dt$$

Since $X_1(t)$ and $dW_1(t)$ are independent, its covariance is zero, and "dt" term is zero as "dt"→0

$$dm_1(t)/dt=a(t)+b_1(t)q_1(t)-2b_1(t)m_1(t) \qquad \cdots\cdots \quad (B-3)$$

$$= a(t)+(v_1+u_1)q_1(t)-2(v_1+u_1)m_1(t)$$

By the same rationale we derive covariance function, $m_{X_1Y_1}(t)$.

From (1,6) substituted by (B-1)

$$X_1(t+dt)=X_1(t)-b_1(t)X_1(t)dt+\sqrt{a(t)+b_1(t)q_1(t)}\,dW_1(t) \qquad \ldots\ldots \quad (B-4)$$

$$X_2(t+dt)=X_2(t)+u_1(t)X_1(t)dt-b_2(t)X_2(t)dt$$
$$+\sqrt{u_1(t)q_1(t)+b_2(t)q_2(t)}\,dW_2(t) \qquad \ldots\ldots \quad (B-5)$$

Take the product (B-4) and (B-5) and get $dm_{12}(t)/dt$ by the same rationale which we had in (B-3)

Since

$X_1(t)$ and $dW_1(t)$

$X_1(t)$ and $dW_2(t)$

$X_2(t)$ and $dW_1(t)$

$X_2(t)$ and $dW_2(t)$

$dW_1(t)$ and $dW_2(t)$

are independent

and $E[X_1(t)dW_1(t)]=0$, etc.

$$dm_{12}(t)/dt=u_1(t)m_1(t)-\{b_1(t)+b_2(t)\}m_{12}(t) \qquad \ldots\ldots \quad (B-6)$$

Appendix C (derivation)

From (3,2) substituted by (3,3)

$$d Y_1(t) = a_h(t)\sqrt{Q_0}\left\{\frac{X_1(t)+\sqrt{Q_0}\,g_1(t)}{X_1(t)+Y_1(t)+\sqrt{Q_0}\{g_1(t)+h_1(t)\}} - \frac{X_1(t)/\sqrt{Q_0}+g_1(t)}{k_1}\right\}dt$$

$$-\left\{U_{h_1}(t)Y_1(t)+\sqrt{Q_0}\,U_{h_1}(t)h_1(t)+U_{h_1}(t)Y_1(t)+\sqrt{Q_0}\,U_{h_1}(t)h_1(t)\right\}dt$$

$$-\sqrt{Q_0}\,dh_1(t) - \sqrt{U_{h_1}(t)\{Y_1(t)/\sqrt{Q_0}+h_1(t)\}}\,dW_{V_{h_1}}(t)$$

$$-\sqrt{U_{h_1}(t)\{Y_1(t)/\sqrt{Q_0}+h_1(t)\}}\,dW_{U_{h_1}}(t)$$

$$+\sqrt{a_h(t)\left\{\frac{X_1(t)+\sqrt{Q_0}\,g_1(t)}{X_1(t)+Y_1(t)+\sqrt{Q_0}\{g_1(t)+h_1(t)\}} - \frac{X_1(t)/\sqrt{Q_0}+g_1(t)}{k_1}\right\}}\,dW_{a_h}(t)$$

..... (C-1)

Since the term

$$\frac{1}{X_1(t)+Y_1(t)+\sqrt{Q_0}\{g_1(t)+h_1(t)\}} = \frac{1}{\sqrt{Q_0}\{g_1(t)+h_1(t)\}}$$

$$- \frac{X_1(t)+Y_1(t)}{Q_0\{g_1(t)+h_1(t)\}^2}$$

$$+ \frac{\{X_1(t)+Y_1(t)\}^2}{Q_0^{3/2}\{g_1(t)+h_1(t)\}^3}$$

$$\vdots$$

..... (C-2)

Substitute (C-2) into (C-1) and take $Q_0 \to \infty$, then, by the same logic ;

$$\frac{dh_1(t)}{dt} = \left\{ \frac{1}{g_1(t) + h_1(t)} - \frac{1}{k_1} \right\} a_h(t)\, g_1(t) - \left\{ v_{h_1}(t) + u_{h_1}(t) \right\} h_1(t)$$

and

$$dY_1(t) = \left\{ \frac{1}{g_1(t) + h_1(t)} - \frac{1}{k_1} \right\} a_h(t)\, X_1(t)\, dt$$

$$- \left\{ v_{h_1}(t) + u_{h_1}(t) \right\} Y_1(t)\, dt$$

$$- \frac{a_h(t)\, g_1(t) \left\{ X_1(t) + Y_1(t) \right\}}{\left\{ g_1(t) + h_1(t) \right\}^2}\, dt$$

$$+ \sqrt{a_h(t)\, g_1(t) \left\{ \frac{1}{g_1(t) + h_1(t)} - \frac{1}{k_1} \right\}}\, dW_{a_h}(t)$$

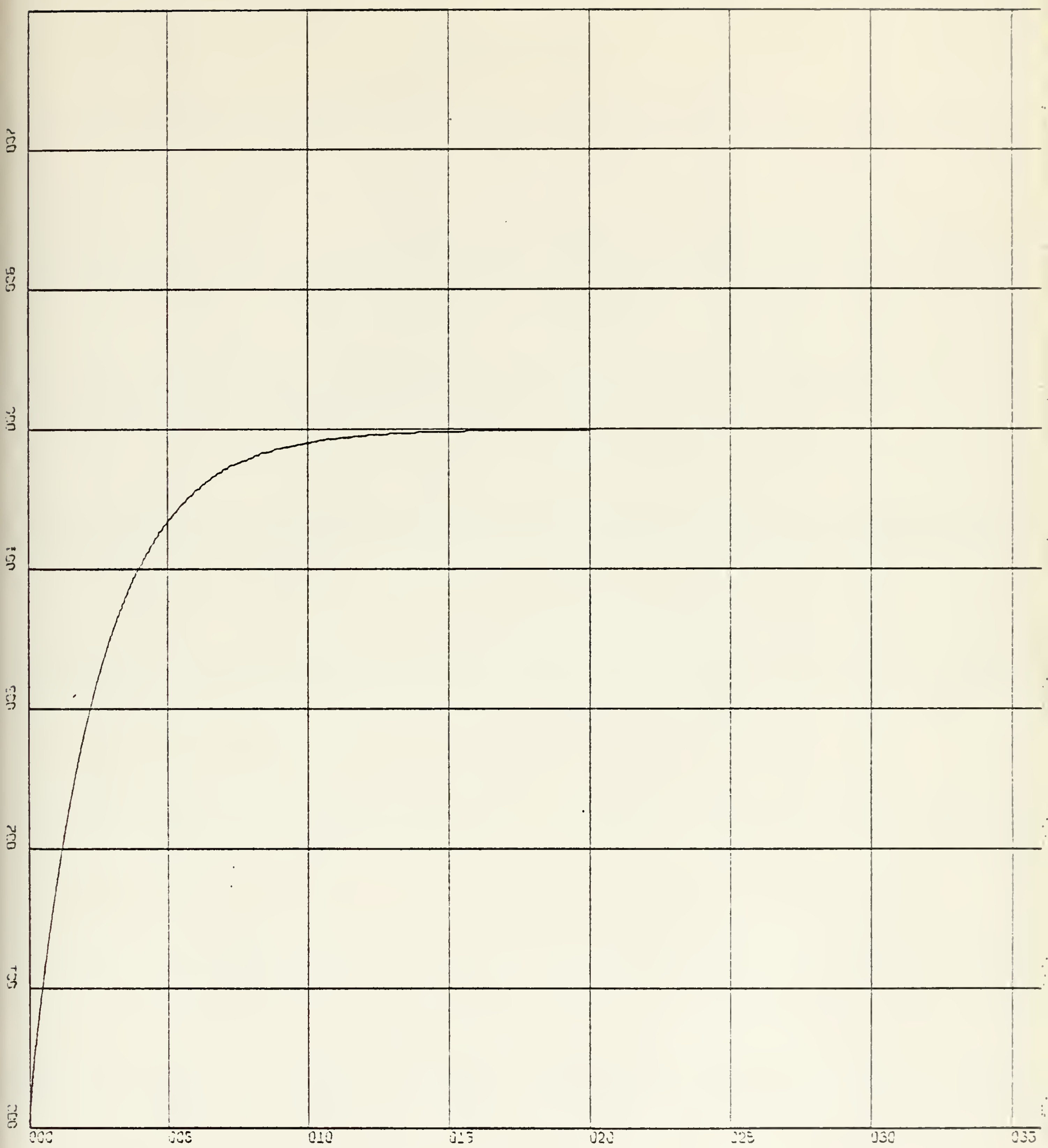
$$- \sqrt{v_{h_1}(t)\, h_1(t)}\, dW_{v_{h_1}}(t)$$

$$- \sqrt{u_{h_1}(t)\, h_1(t)}\, dW_{u_{h_1}}(t)$$

where $\quad g_1(t) + h_1(t) \leq k_1$

The other equations were derived by the same logic.

graph # 1-1

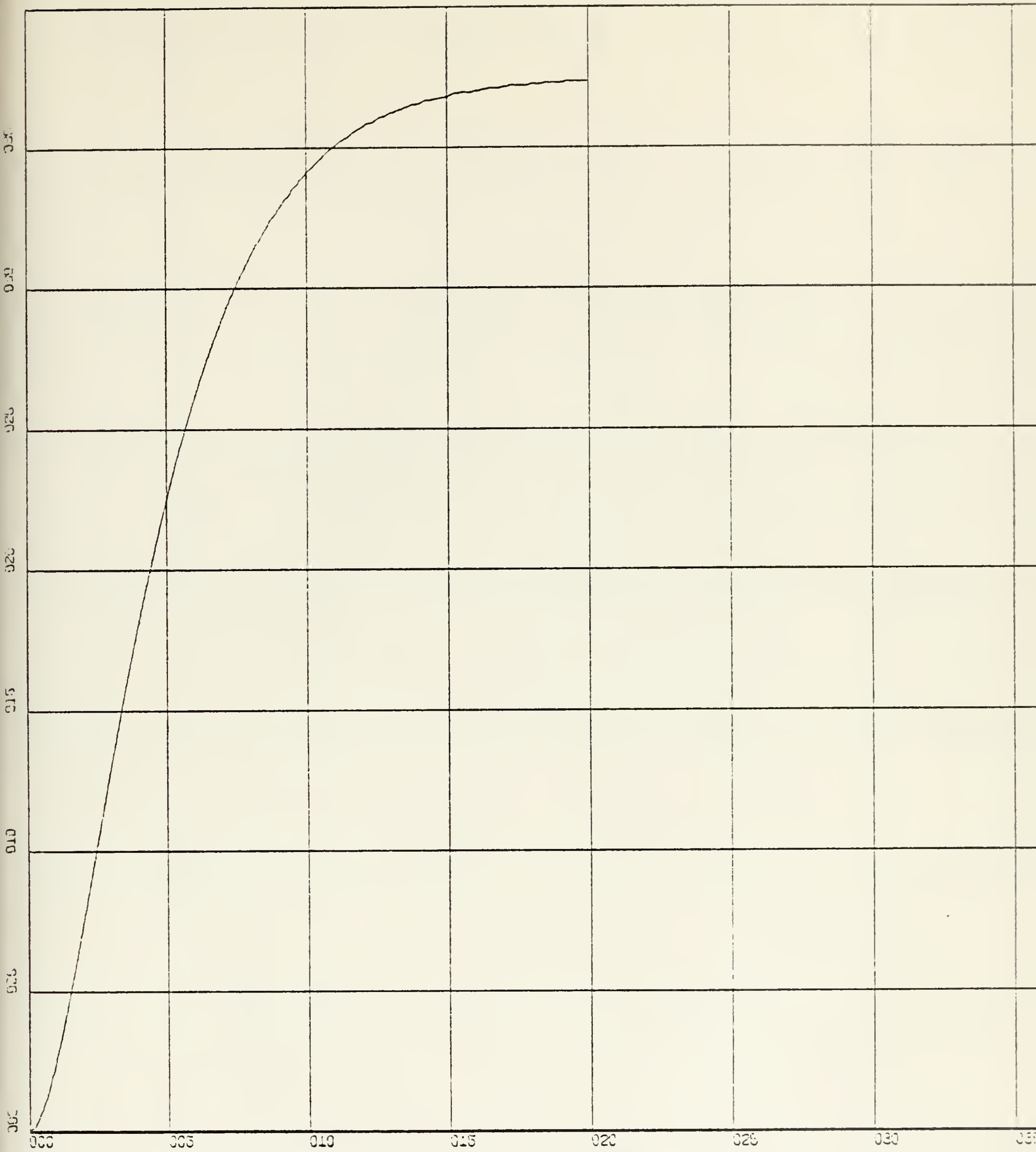


X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=1.00E-03 UNITS INCH.

MODEL-1

RUN 1 Q-1 VS TIME

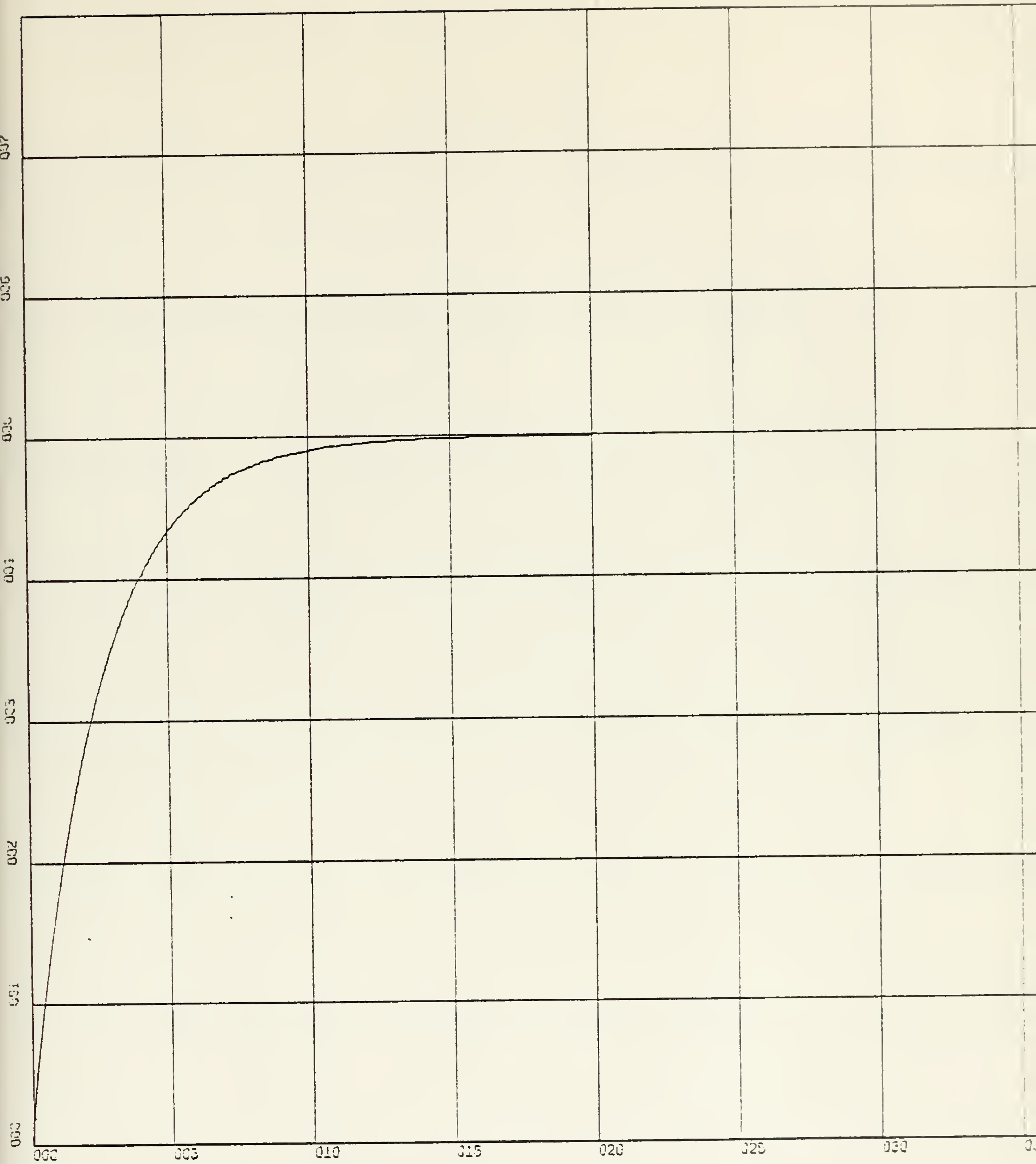
graph # 1-2



X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=5.00E-04 UNITS INCH.

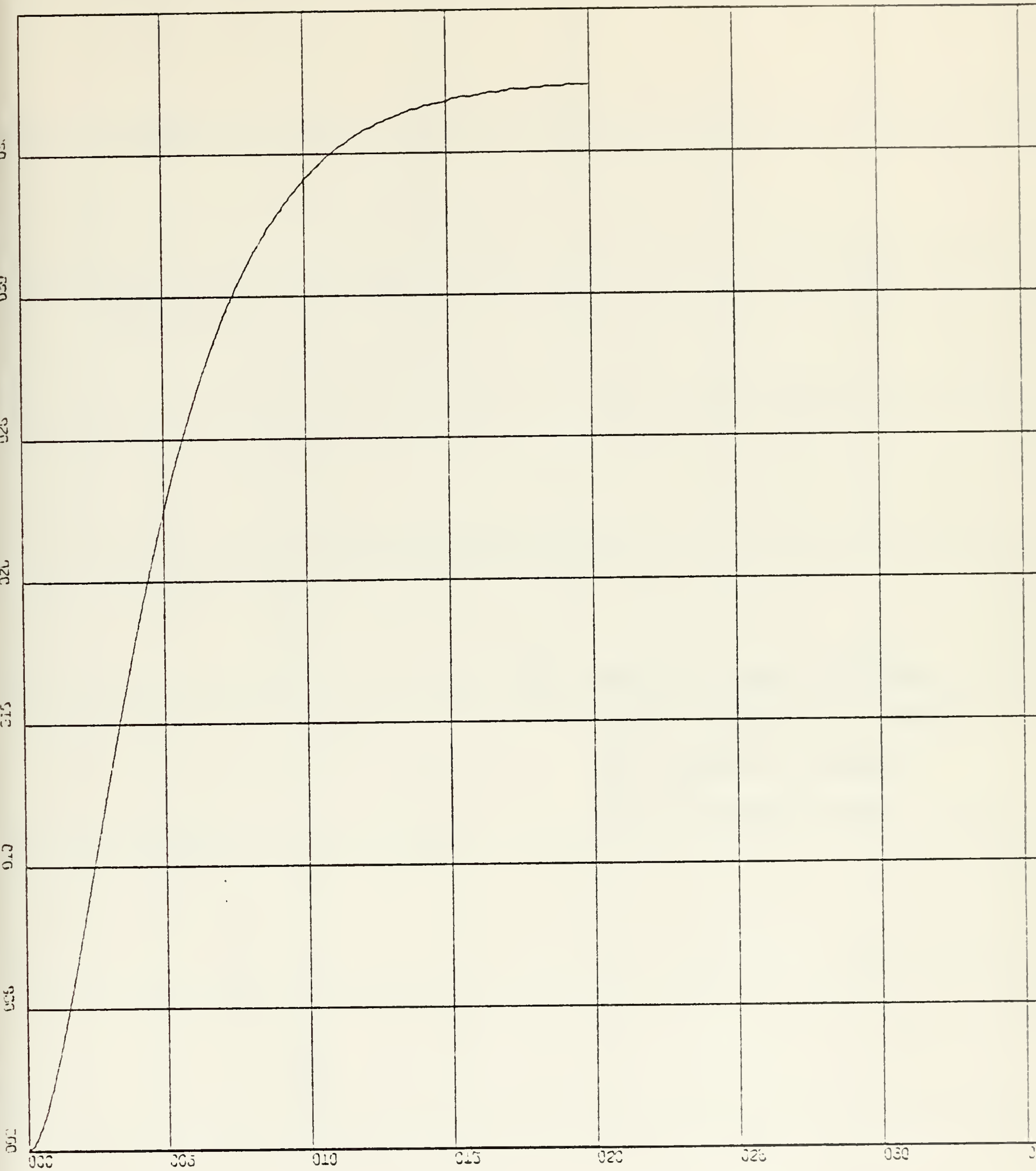
MODEL-1

RUN 1 Q-2 VS TIME



X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=1.00E-03 UNITS INCH.
MODEL-1
RUN 1 M-1 VS TIME

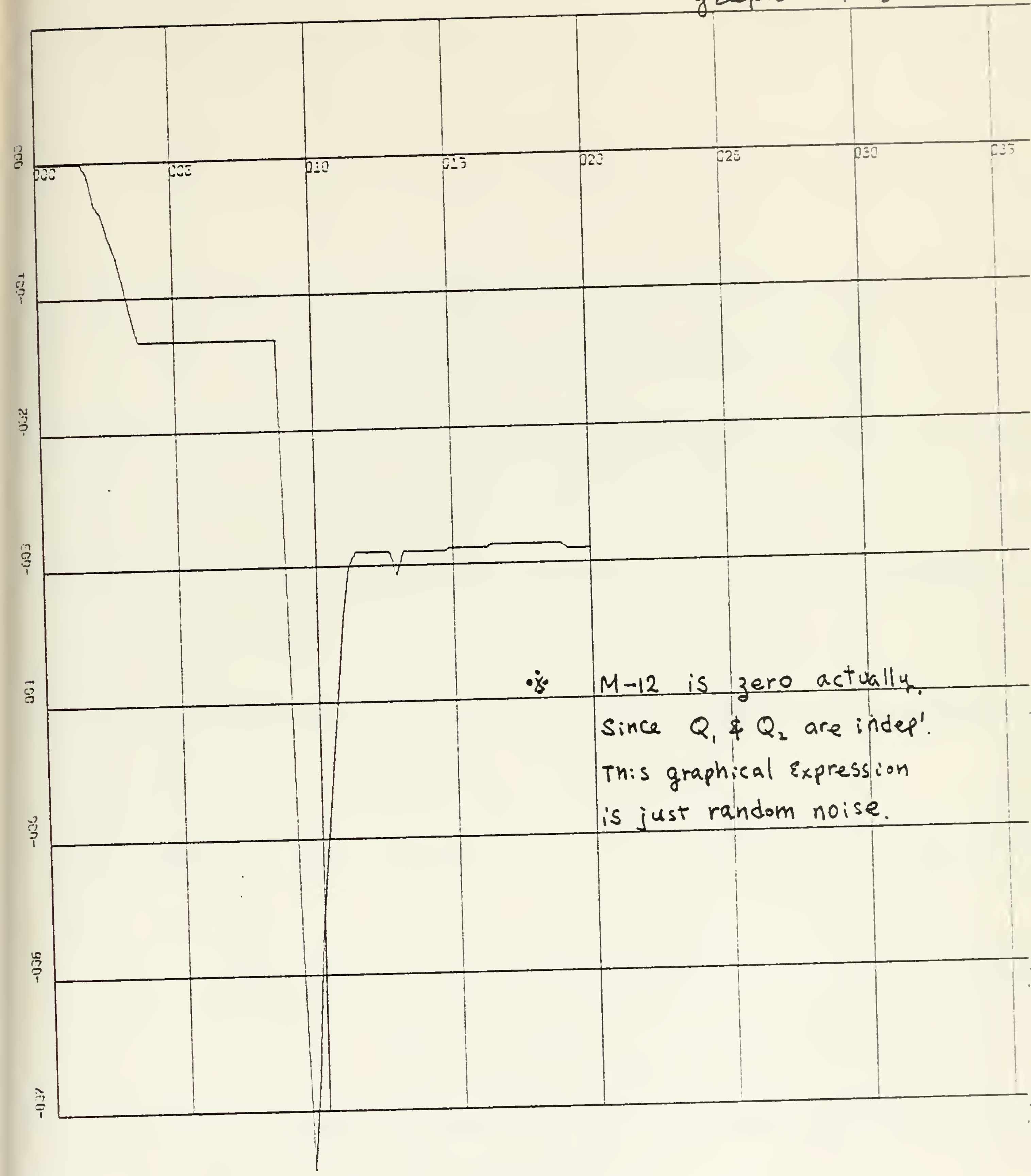
graph #1-4



X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=5.00E-04 UNITS INCH.

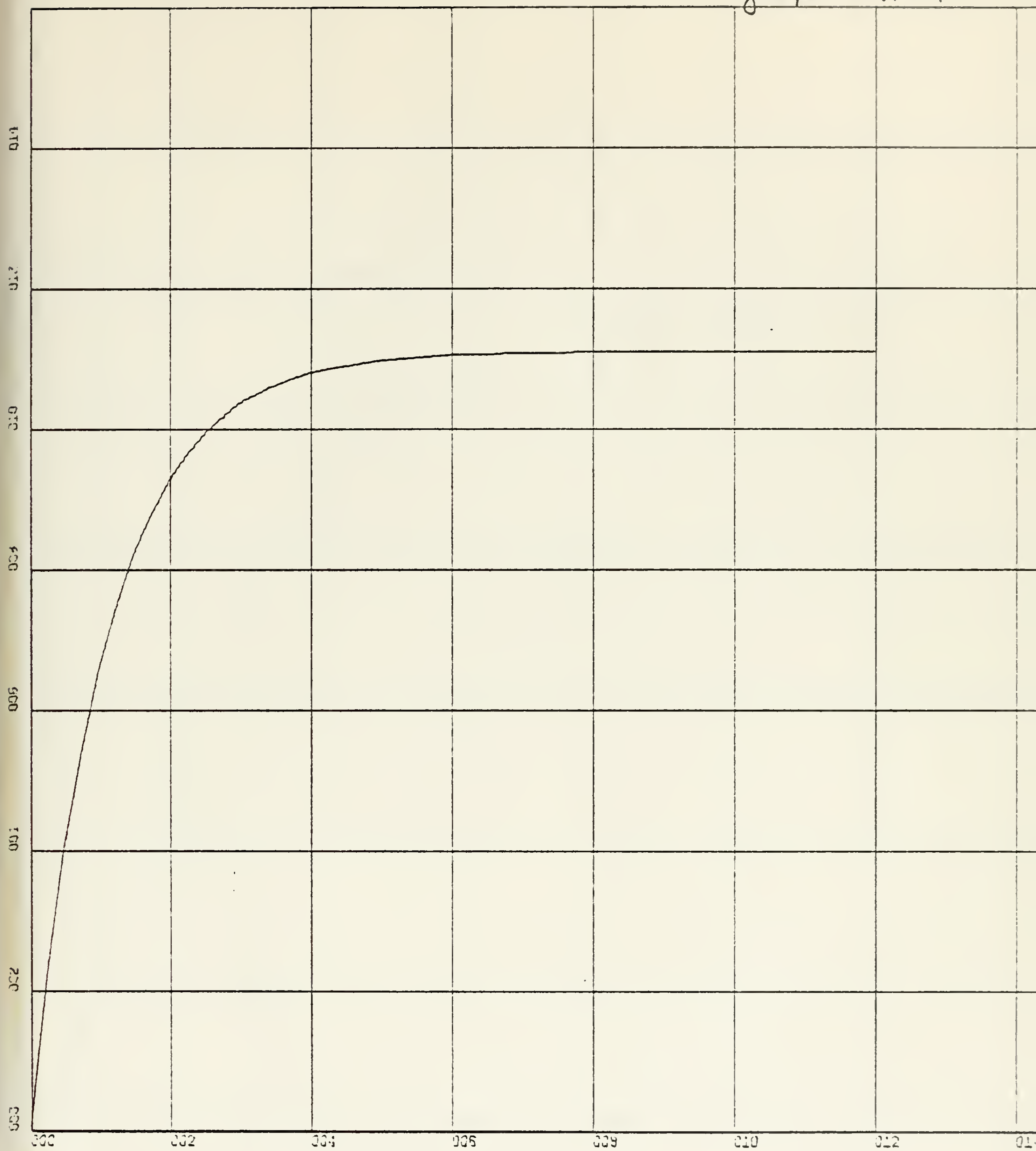
MODEL-1

RUN 1 M-2 VS TIME



X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=1.00E-10 UNITS INCH.
MODEL-1
RUN 1
M-12 VS TIME

Graph # 2-1

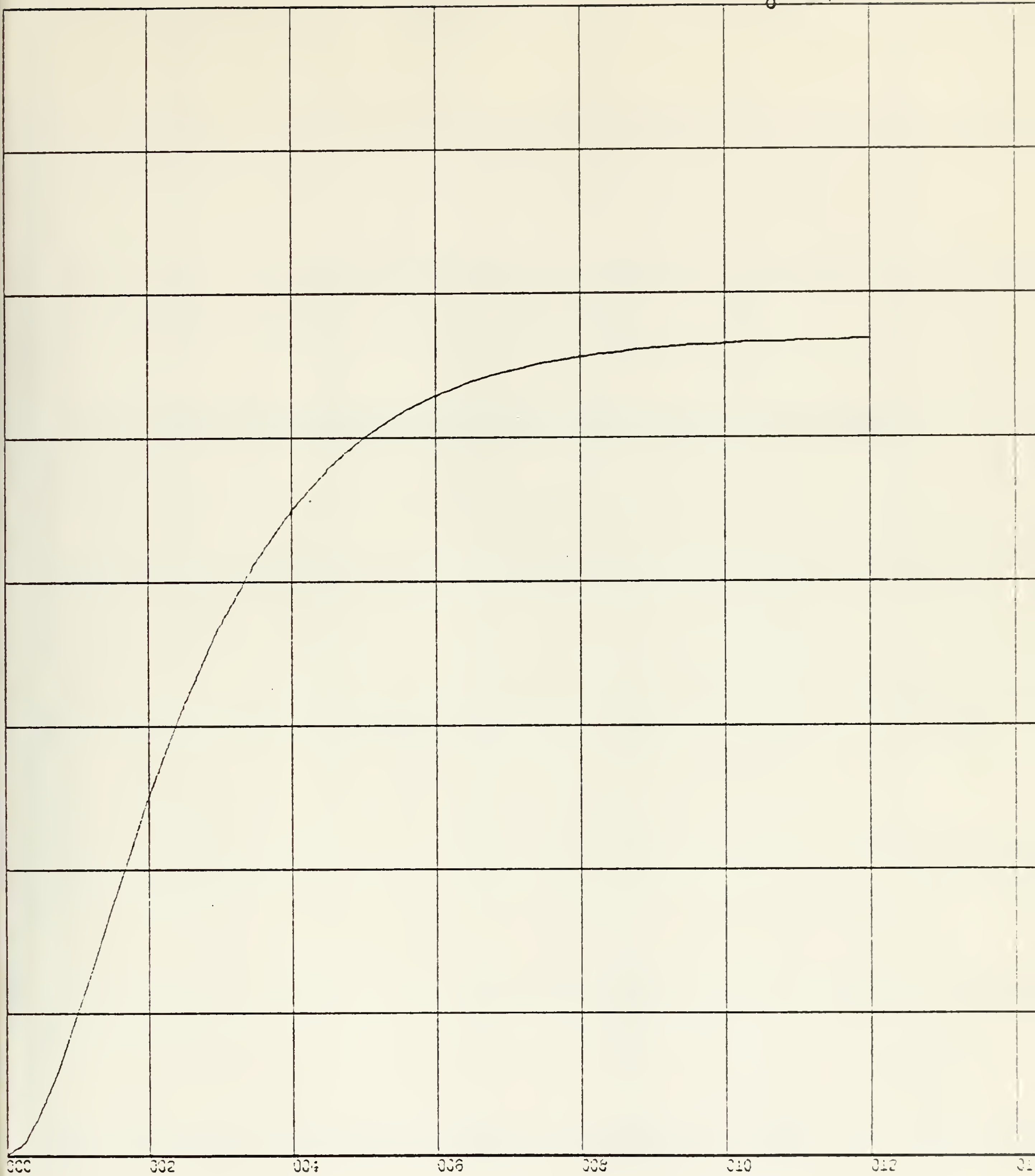


X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

MØDEL-2

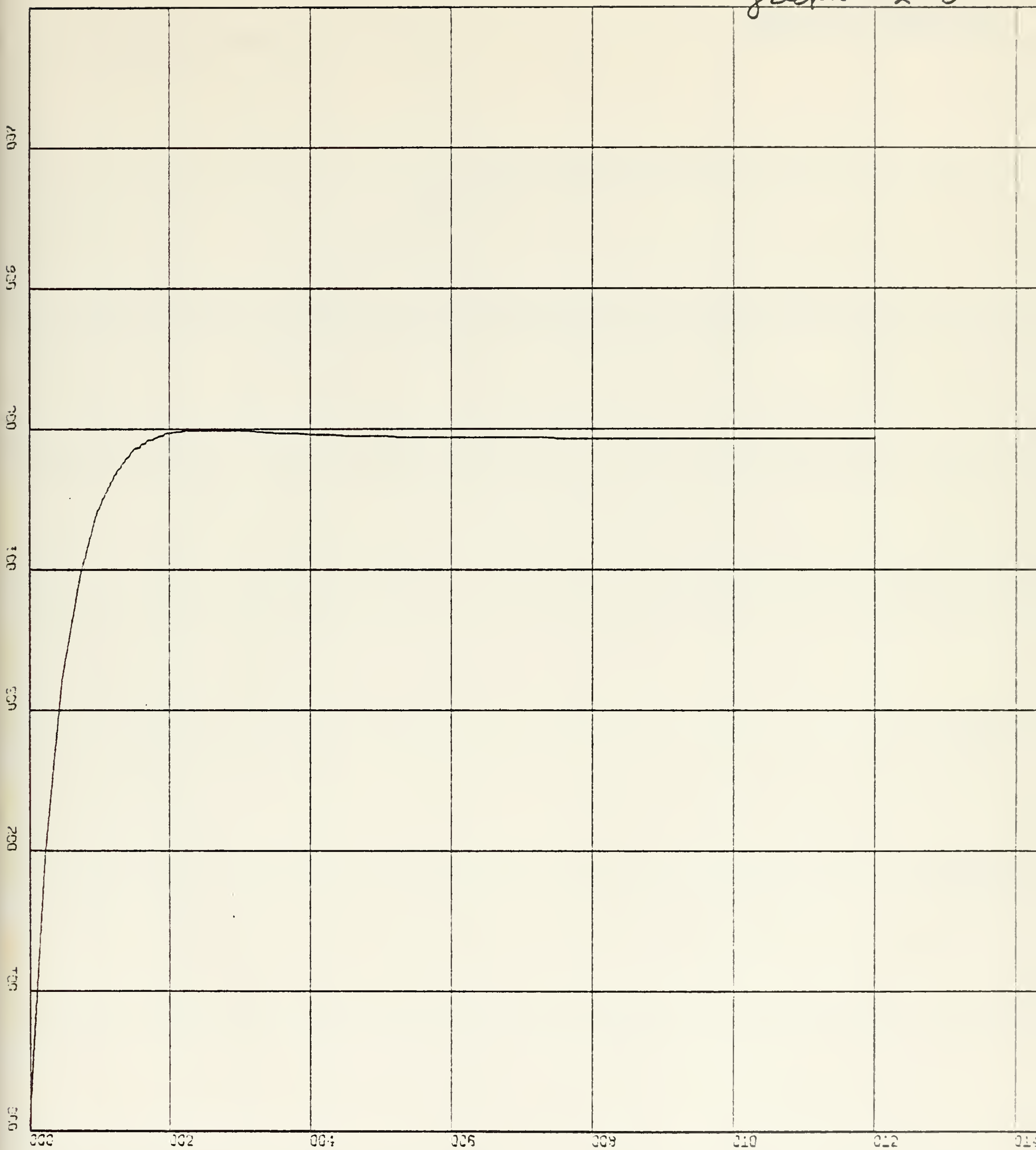
RUN 1 Q-1 VS TIME

graph #2-2



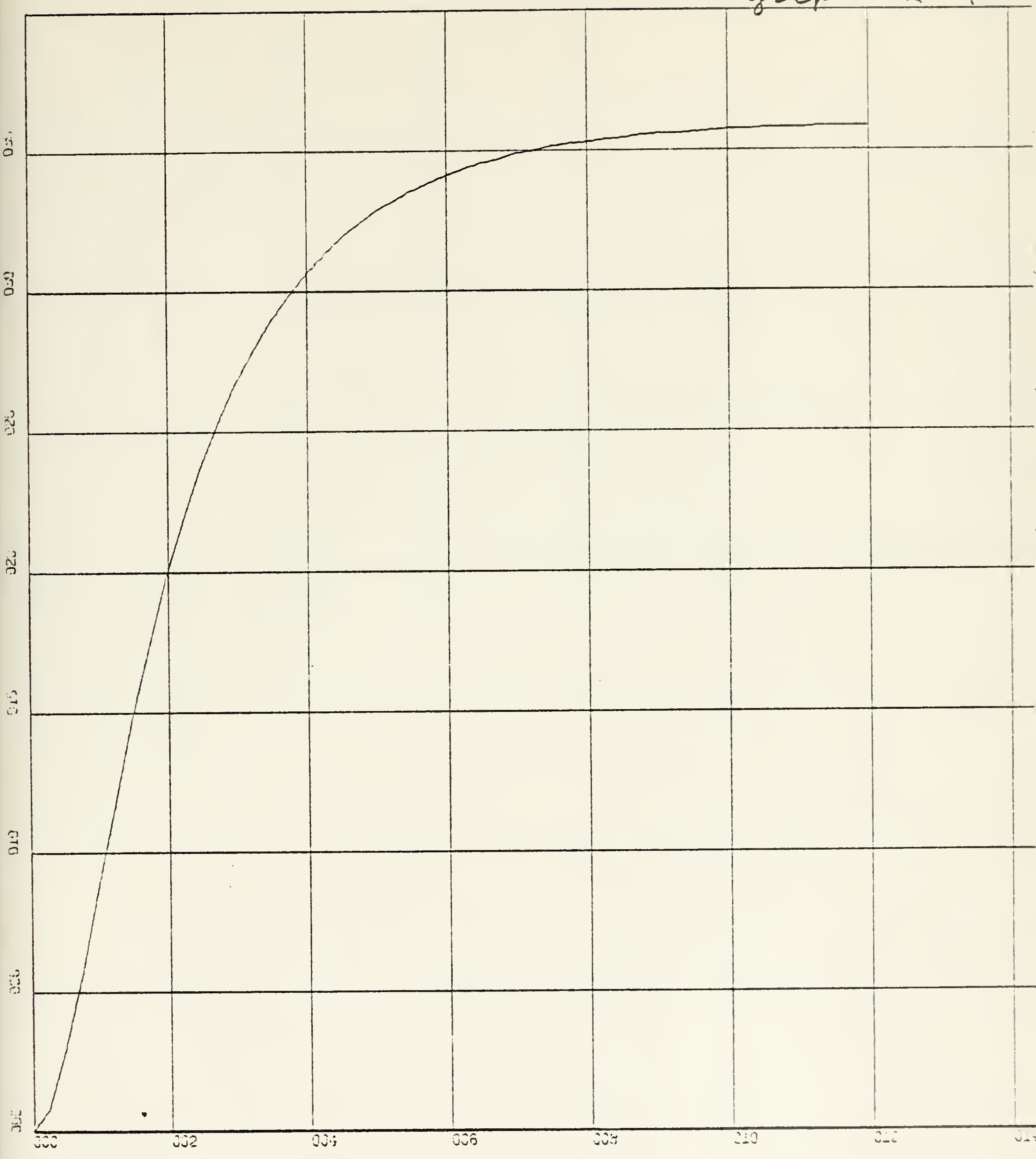
X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=1.00E-04 UNITS INCH.
MODEL-2
RUN 1 Q-2 VS TIME

graph #2-3



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=1.00E-04 UNITS INCH.
MØDEL-2
RUN 1 M-1 VS TIME

graph #2-4



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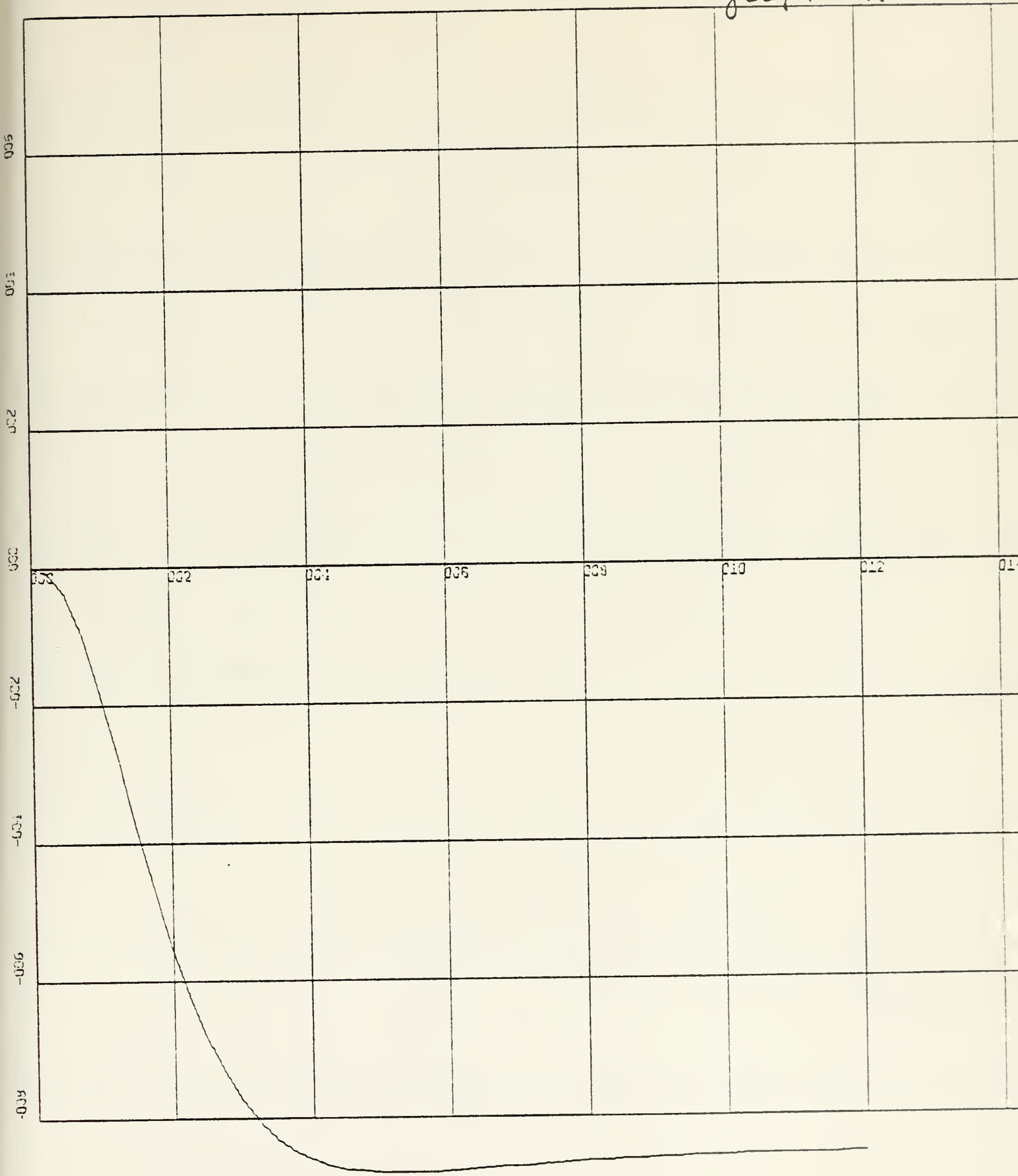
Y-SCALE=5.00E-05 UNITS INCH.
MØDEL-2
RUN 1 M-2 VS TIME

graph # 2-5

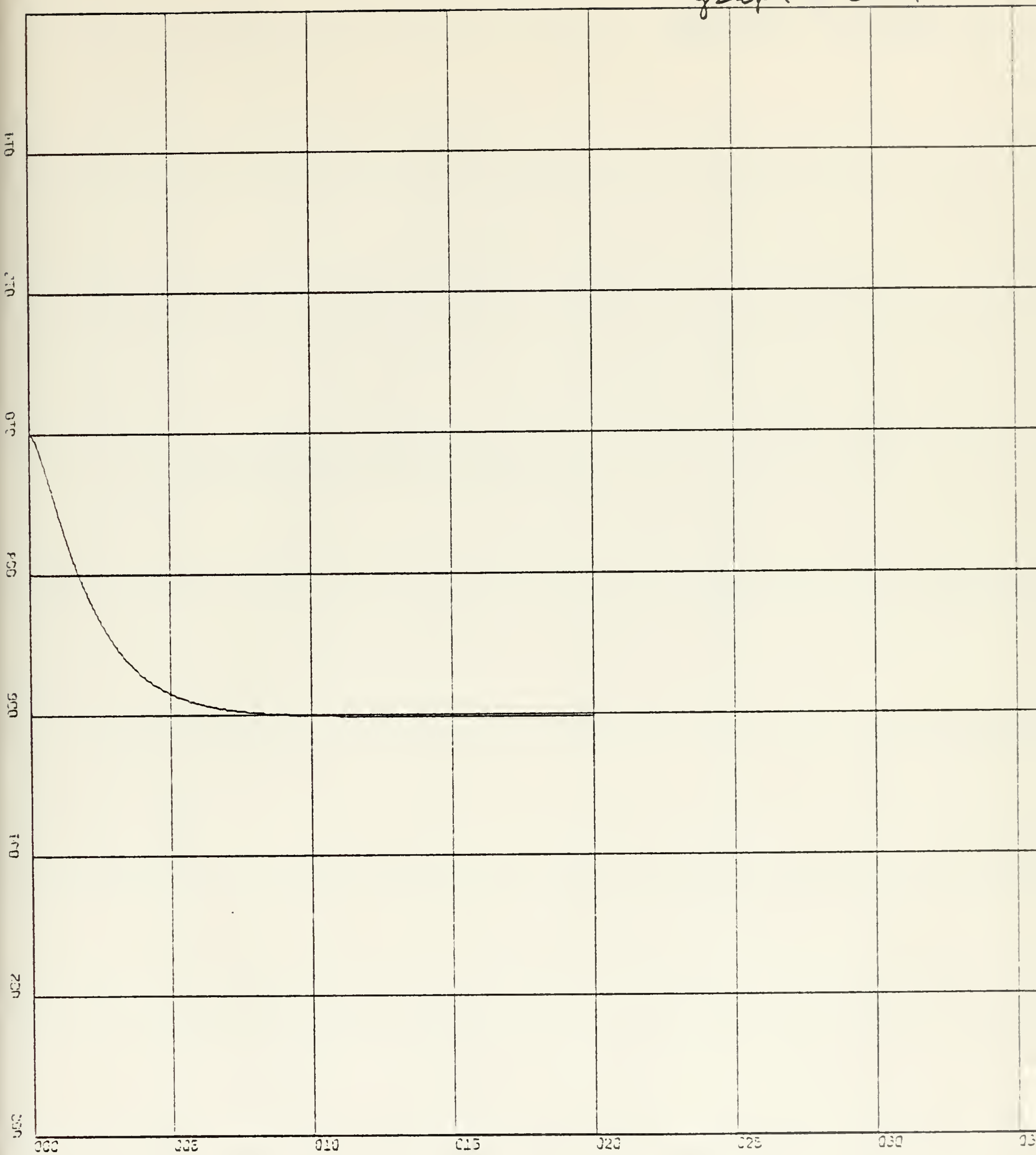


X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-05 UNITS INCH.

MODEL-2

RUN 1 M-12 VS TIME

graph # 3-1

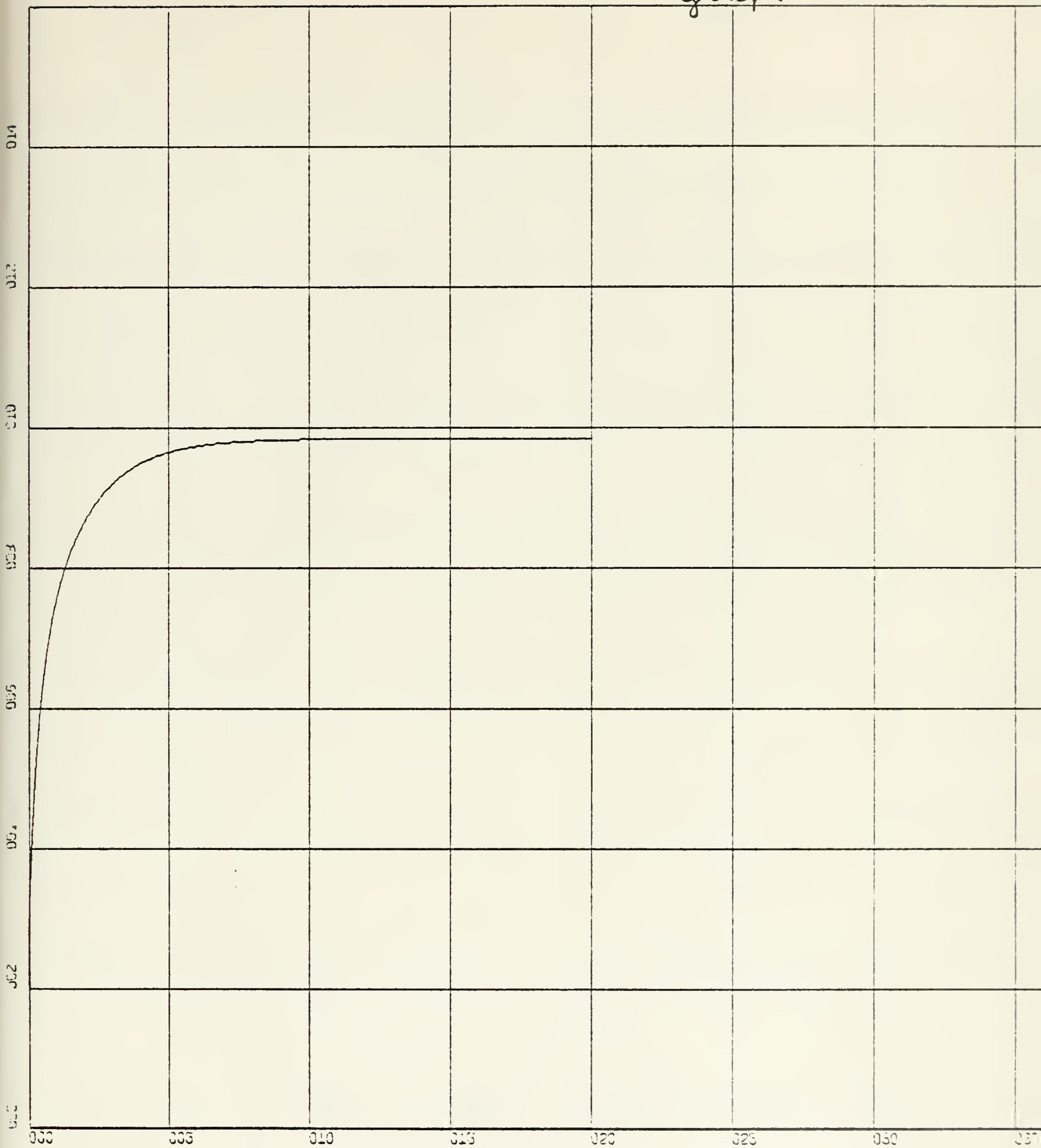


X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

MODEL-3

RUN 1 G-1 VS TIME

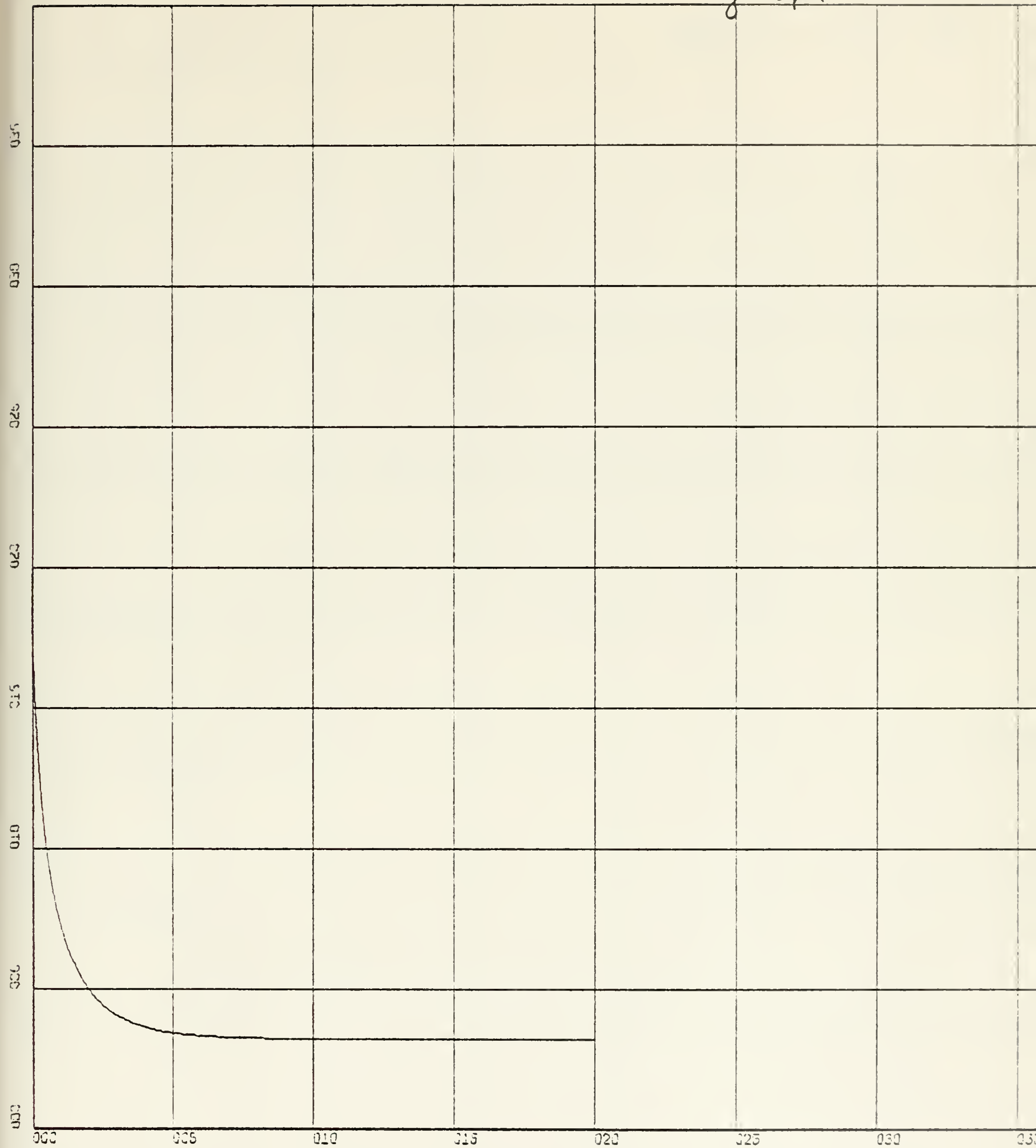
Graph # 3~2



X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

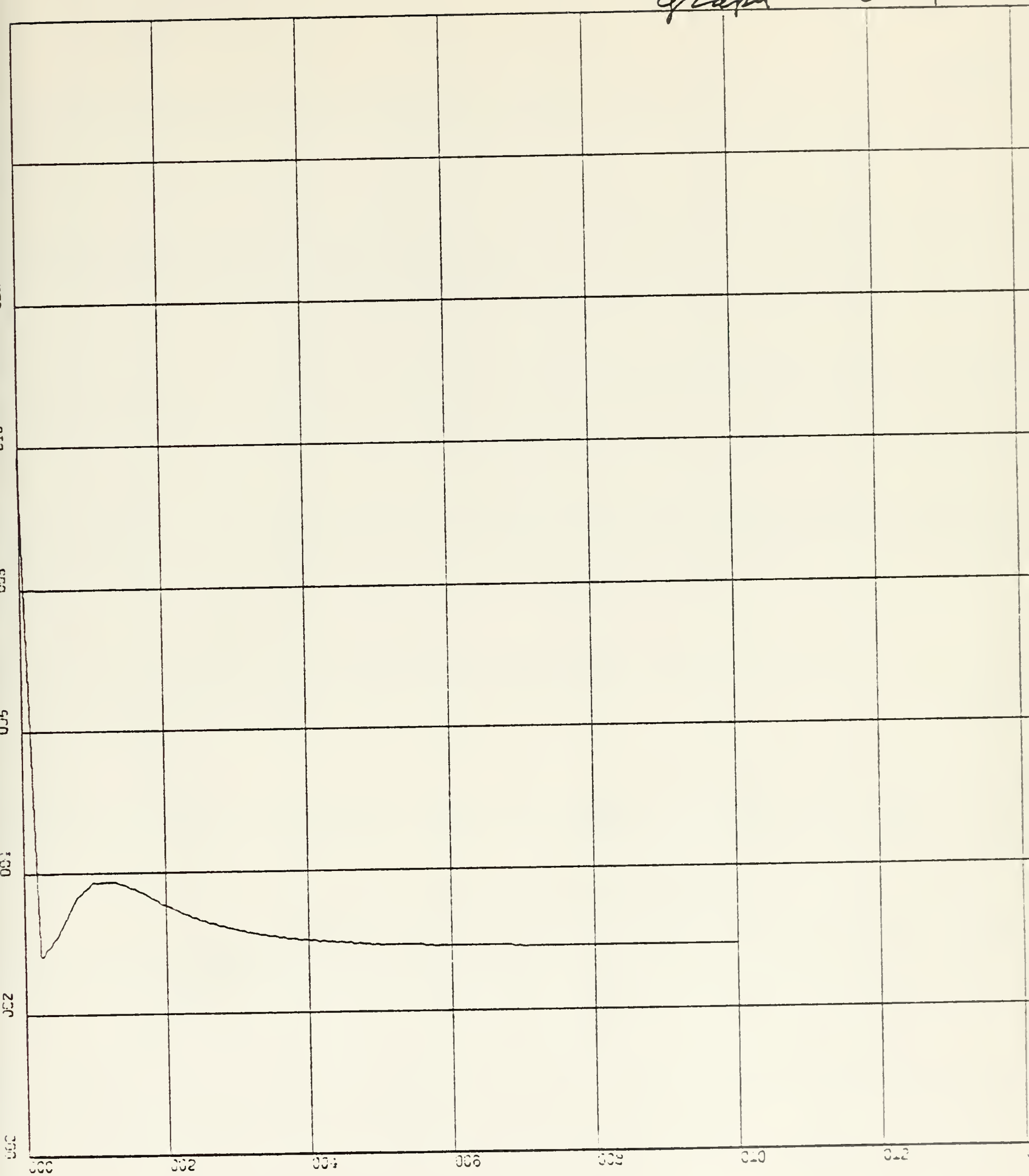
MØDEL-3

RUN 1 H-1 VS TIME



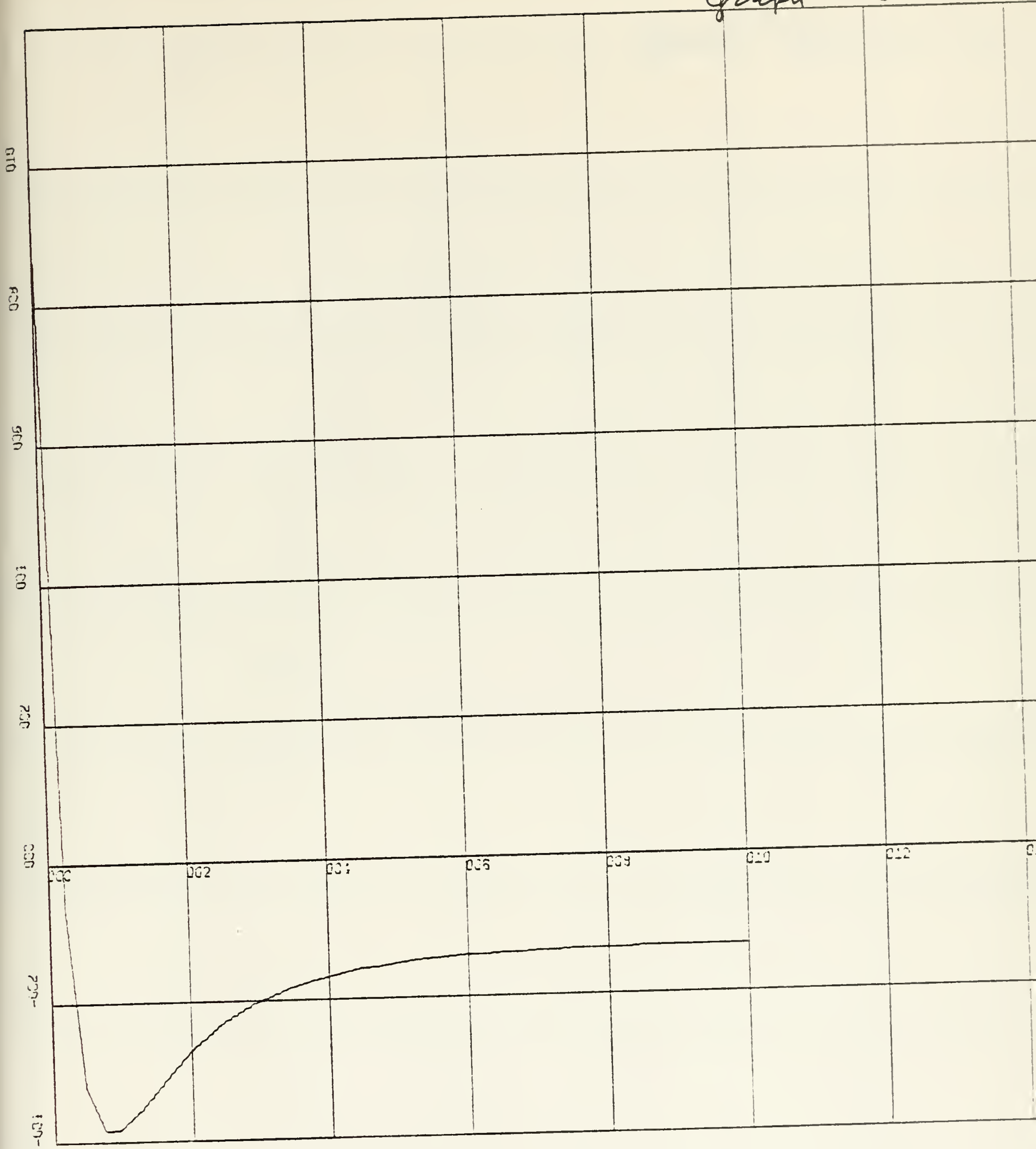
X-SCALE=5.00E+00 UNITS INCH.
Y-SCALE=5.00E-04 UNITS INCH.
MODEL-3
RUN 1 MX-1 VS TIME

graph # 3-4



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.
MØDEL-3
RUN 1 MY-1 VS TIME

graph # 3 − 5



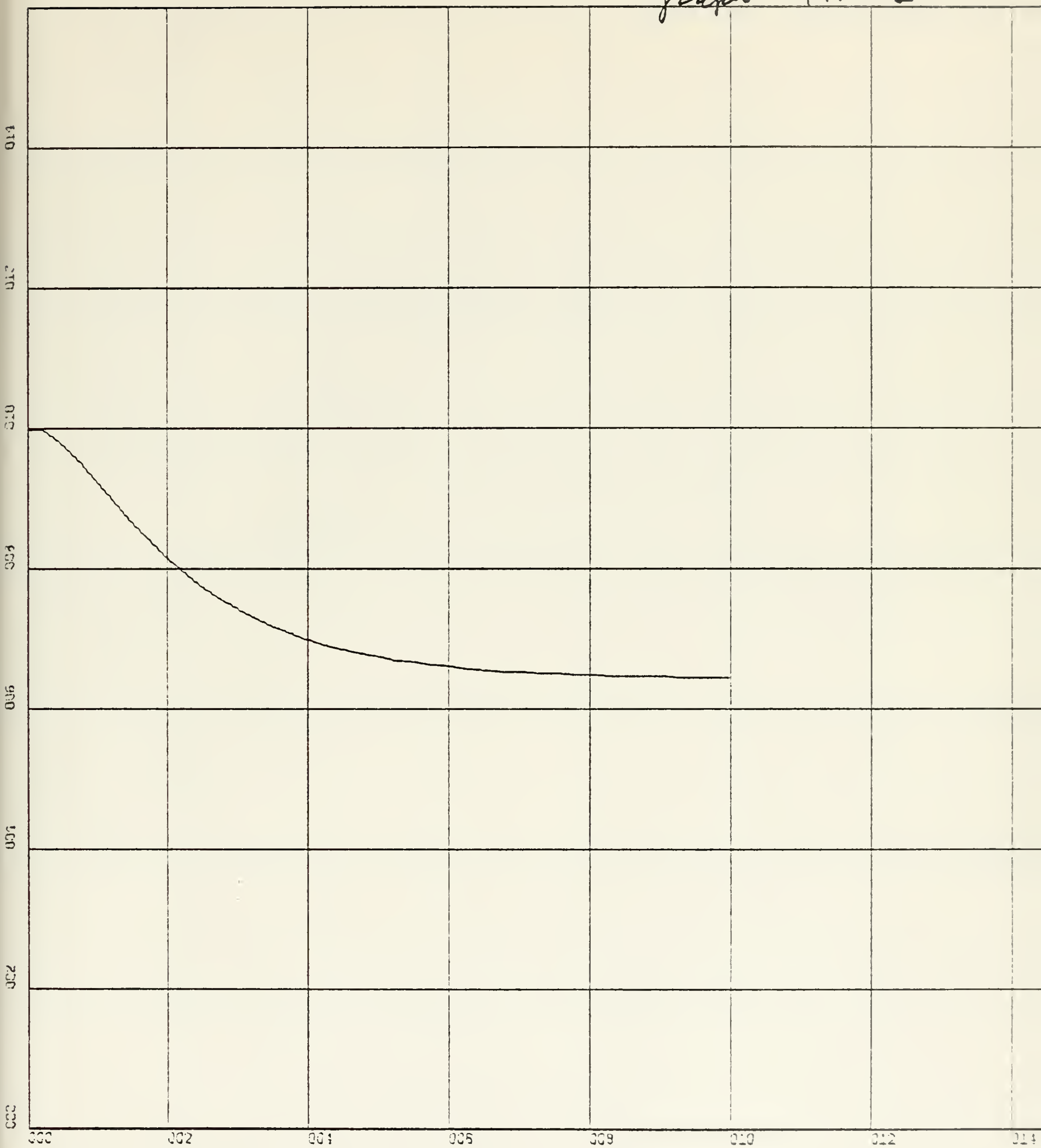
X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

MODEL-3

RUN 1

M-X1*Y1 VS TIME

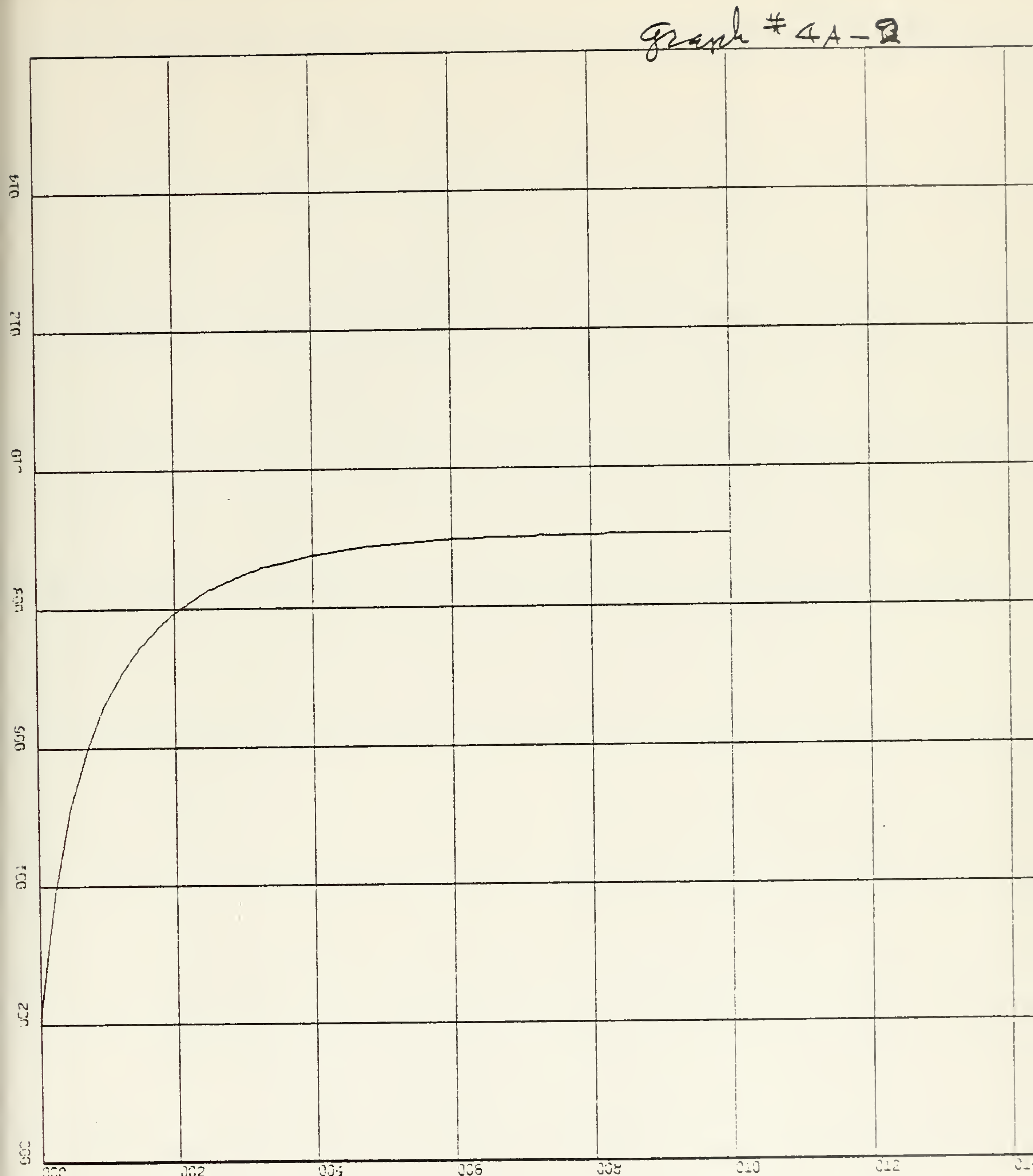
Graph #4A - 1



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

MODEL-4A

RUN 1 G-1 VS TIME

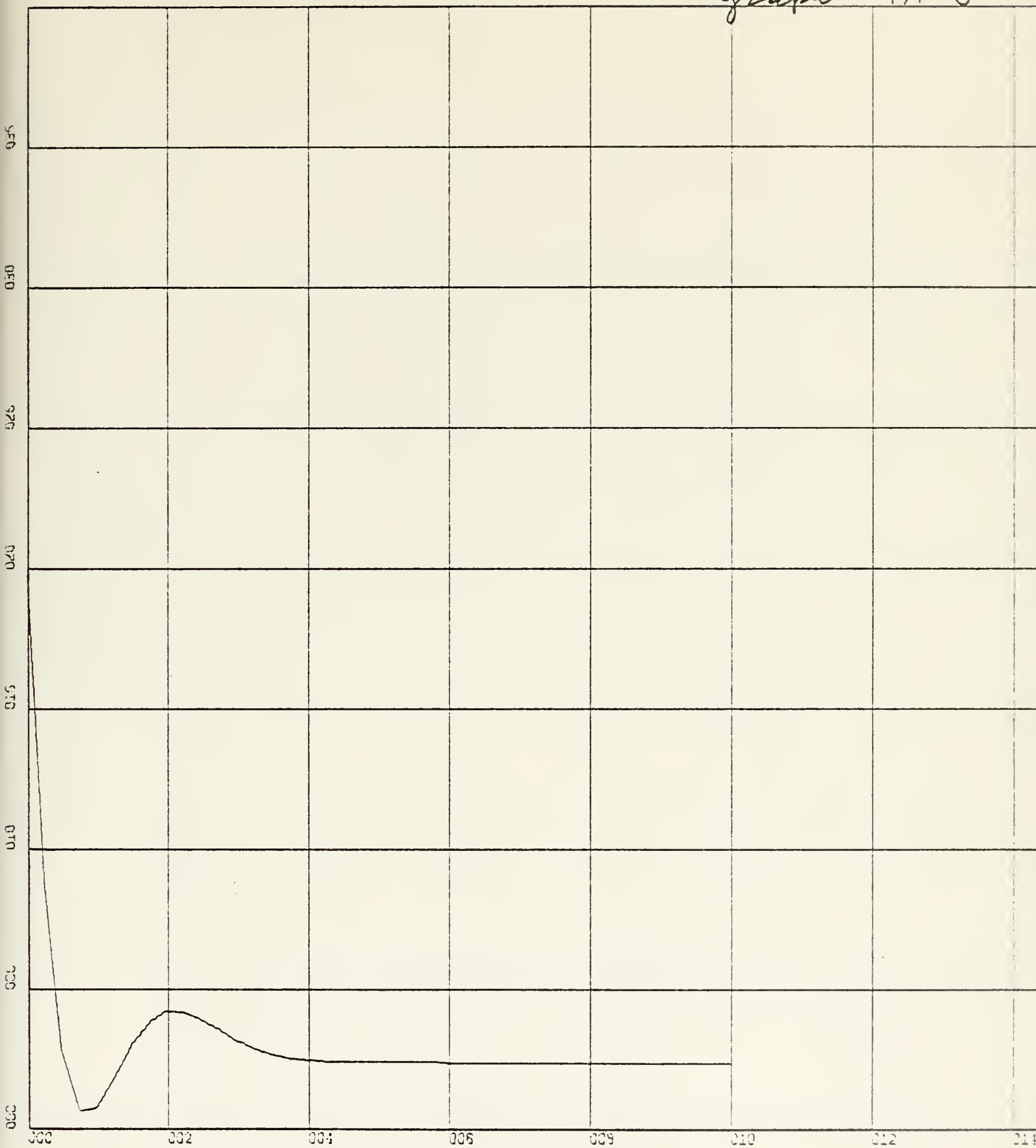


X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.

MØDEL-4A

RUN 1 H-1 VS TIME

graph # 4A-3

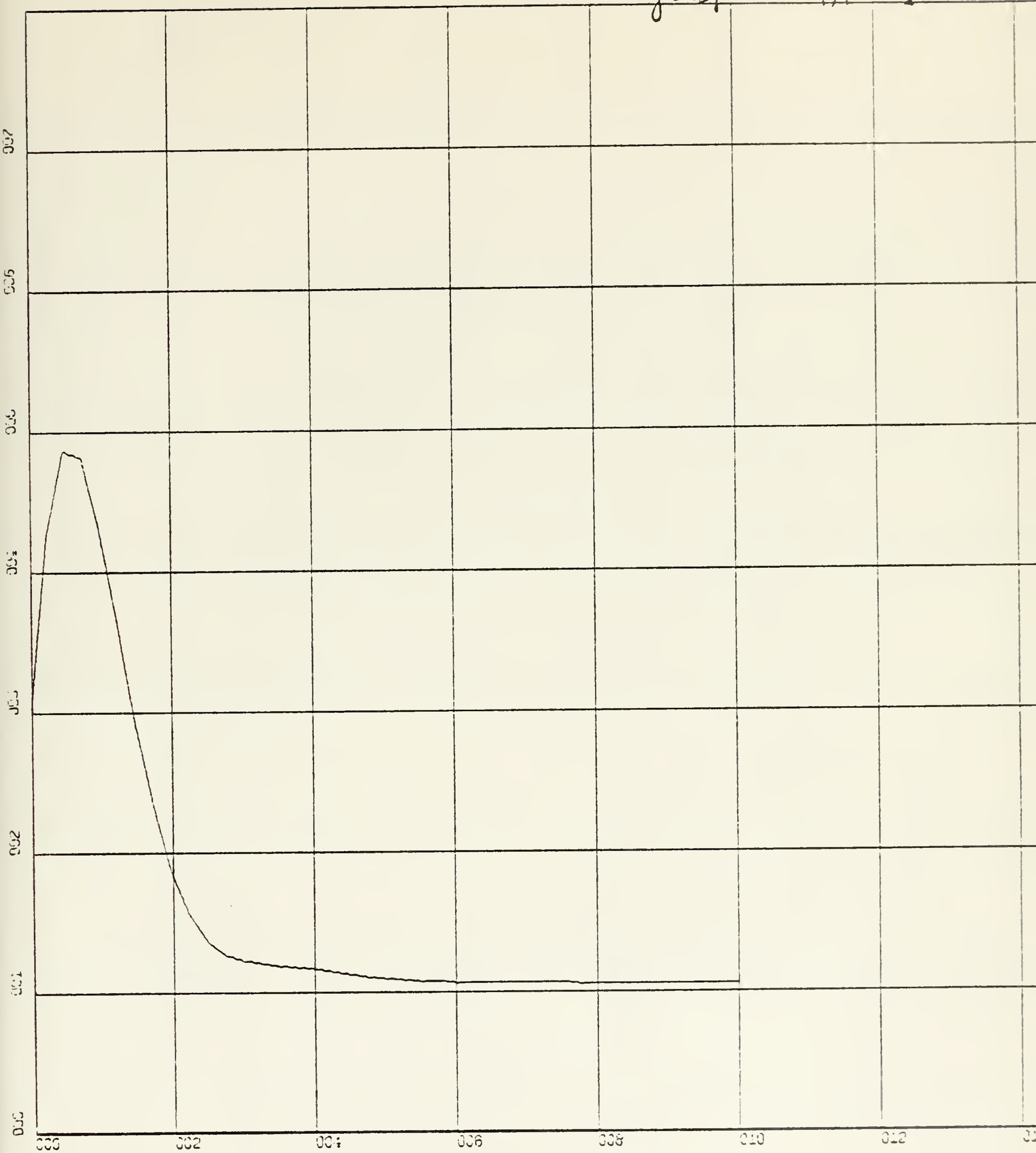


X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=5.00E-04 UNITS INCH.

MØDEL-4A

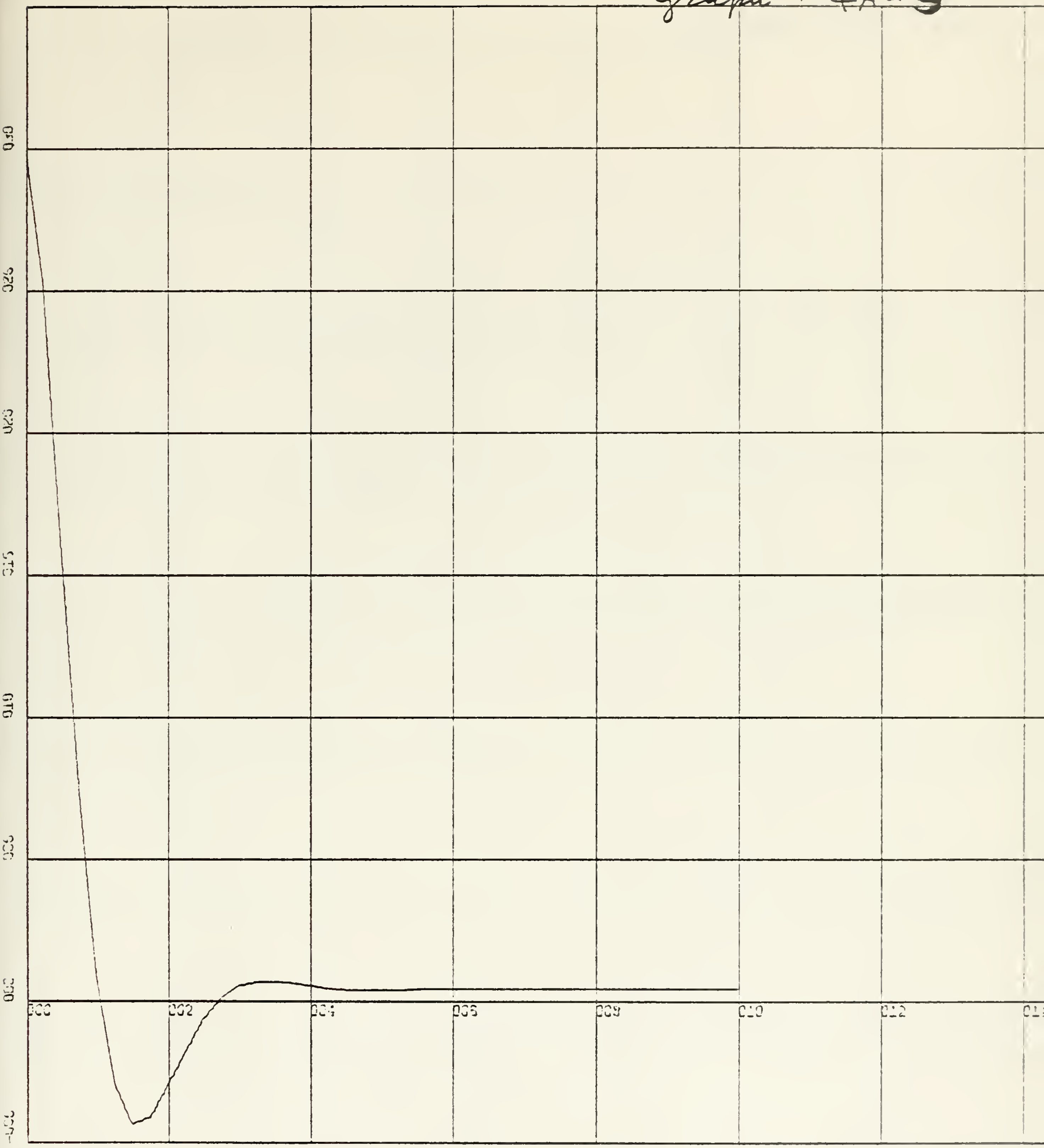
RUN 1 MX-1 VS TIME

graph # 4A-4



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=1.00E-03 UNITS INCH.
MØDEL-4A
RUN 1
MY-1 VS TIME

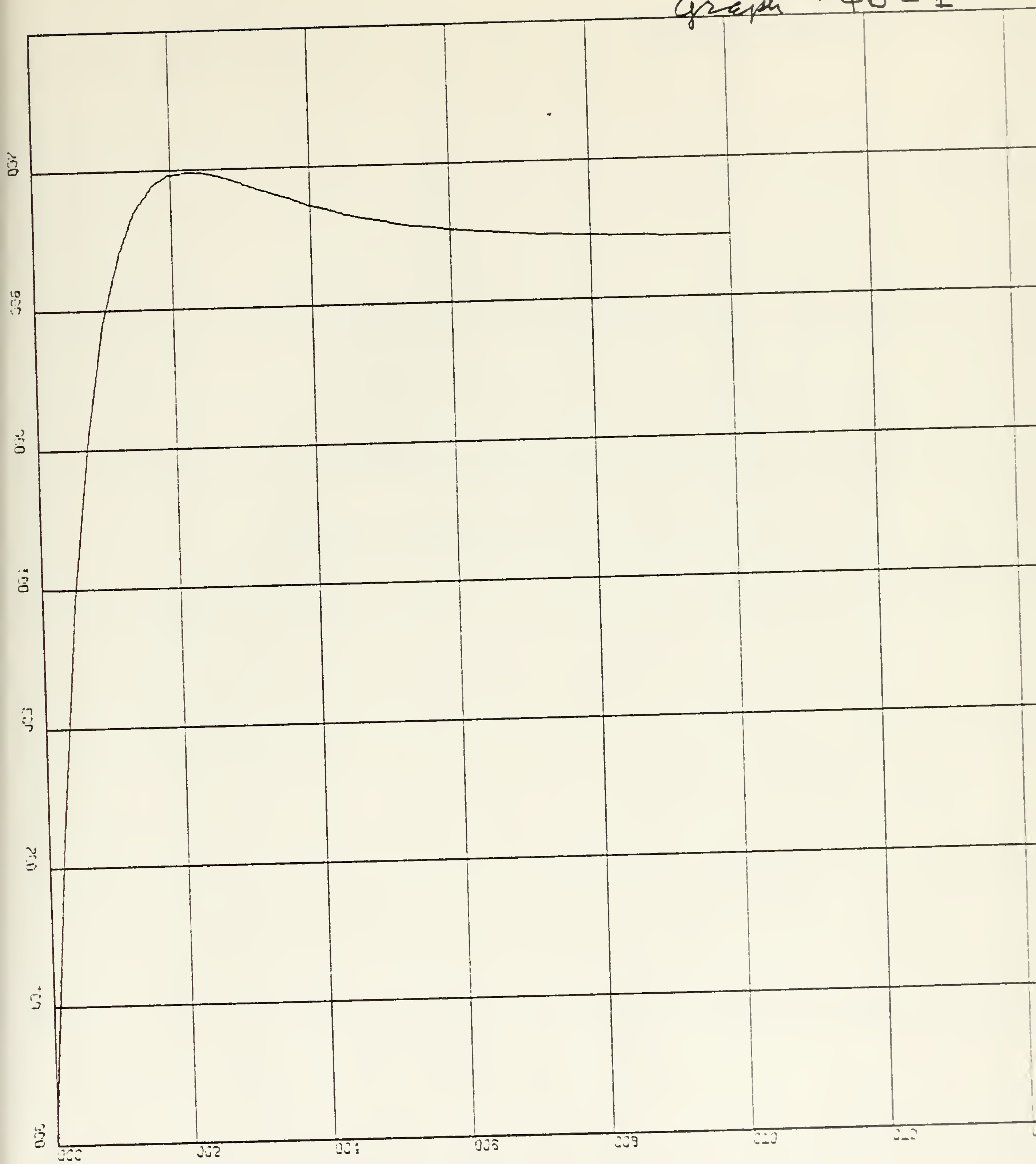
Graph # 4A-5



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=5.00E-04 UNITS INCH.

MODEL-4A

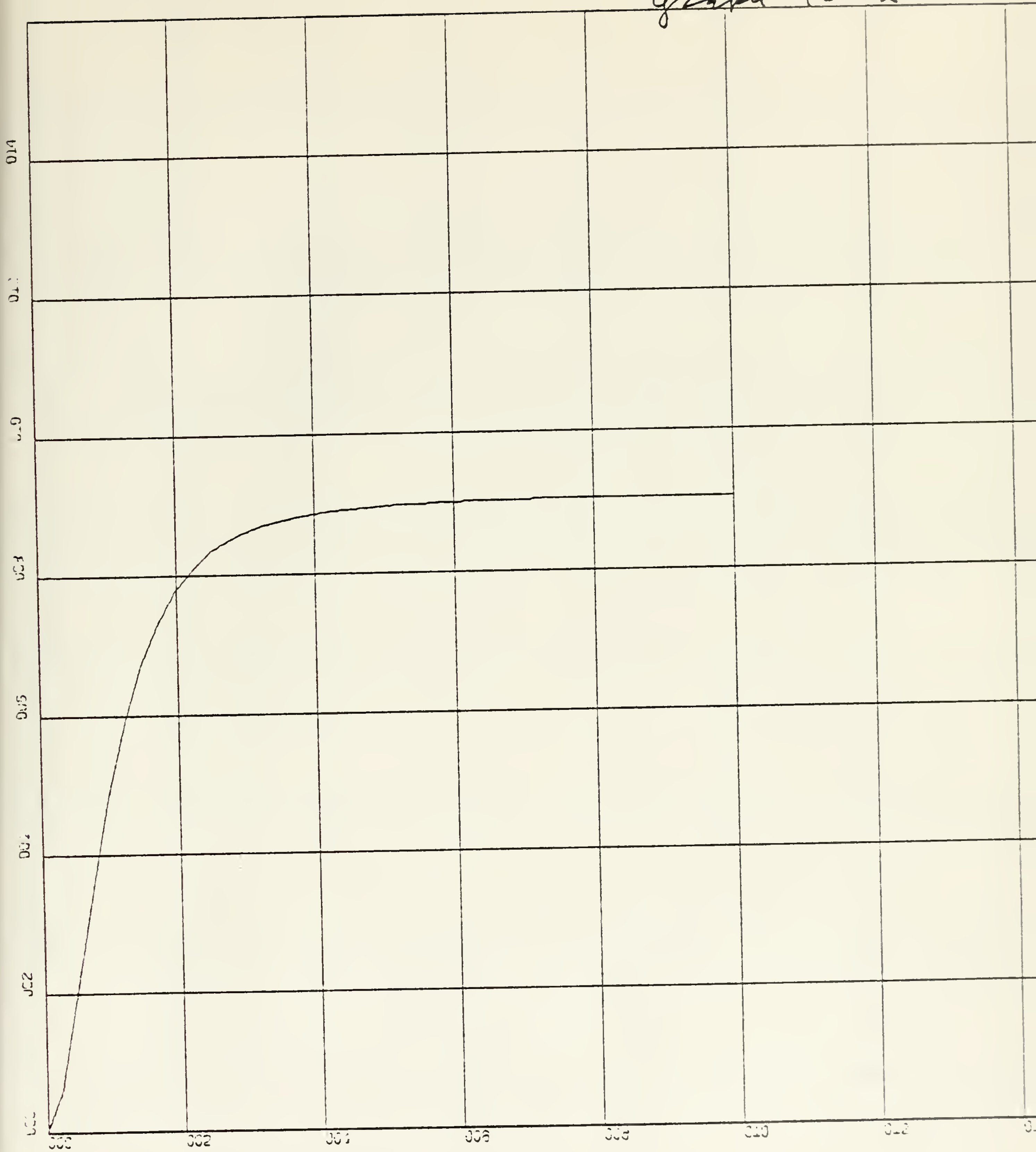
RUN 1 M-X1*Y1 VS TIME



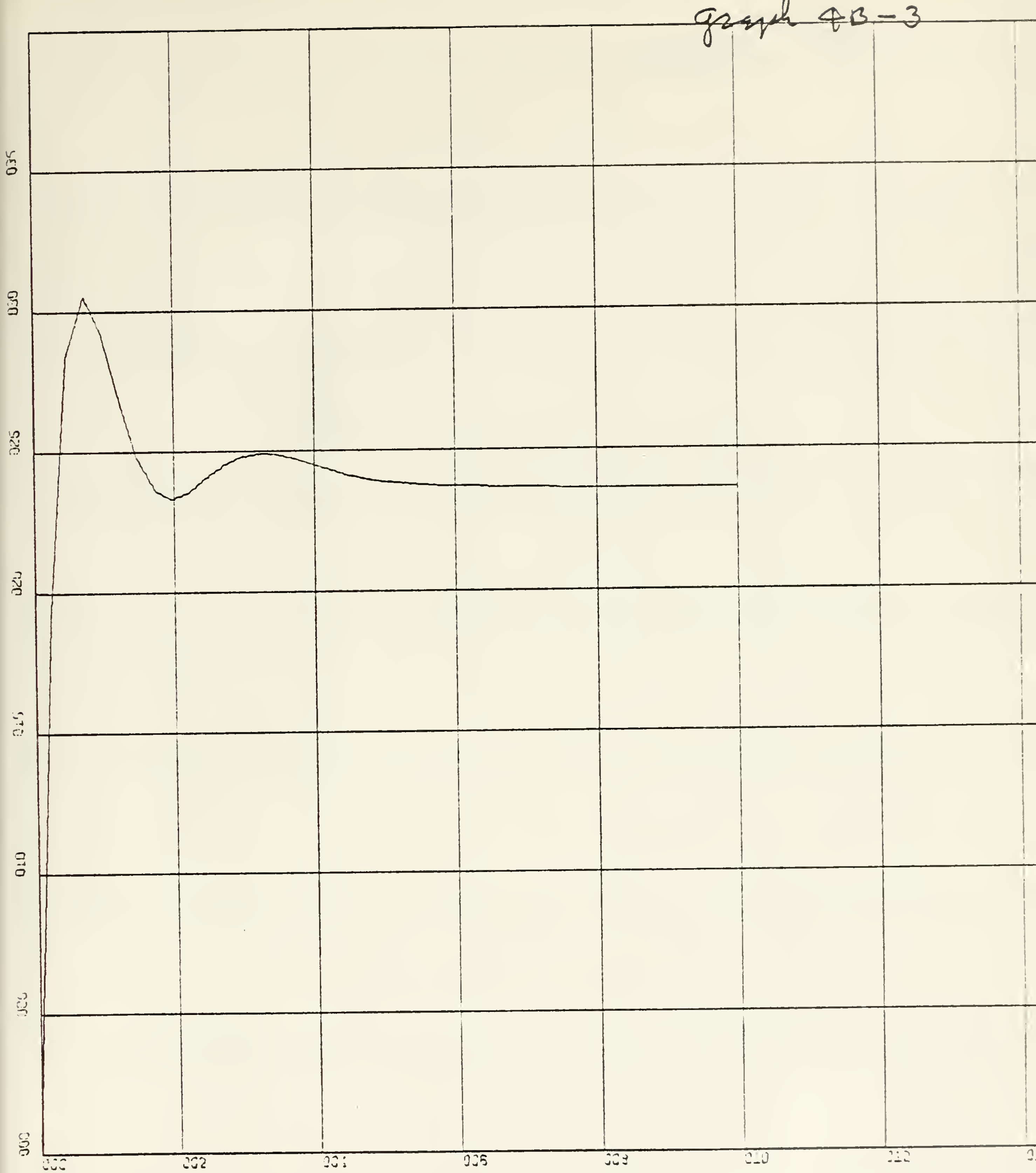
Graph # 4B - 1

X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=1.00E-04 UNITS INCH.
MODEL-4B
RUN 1
G-1 VS TIME

graph 4B-2



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.
MODEL-4B
RUN 1 H-1 VS TIME

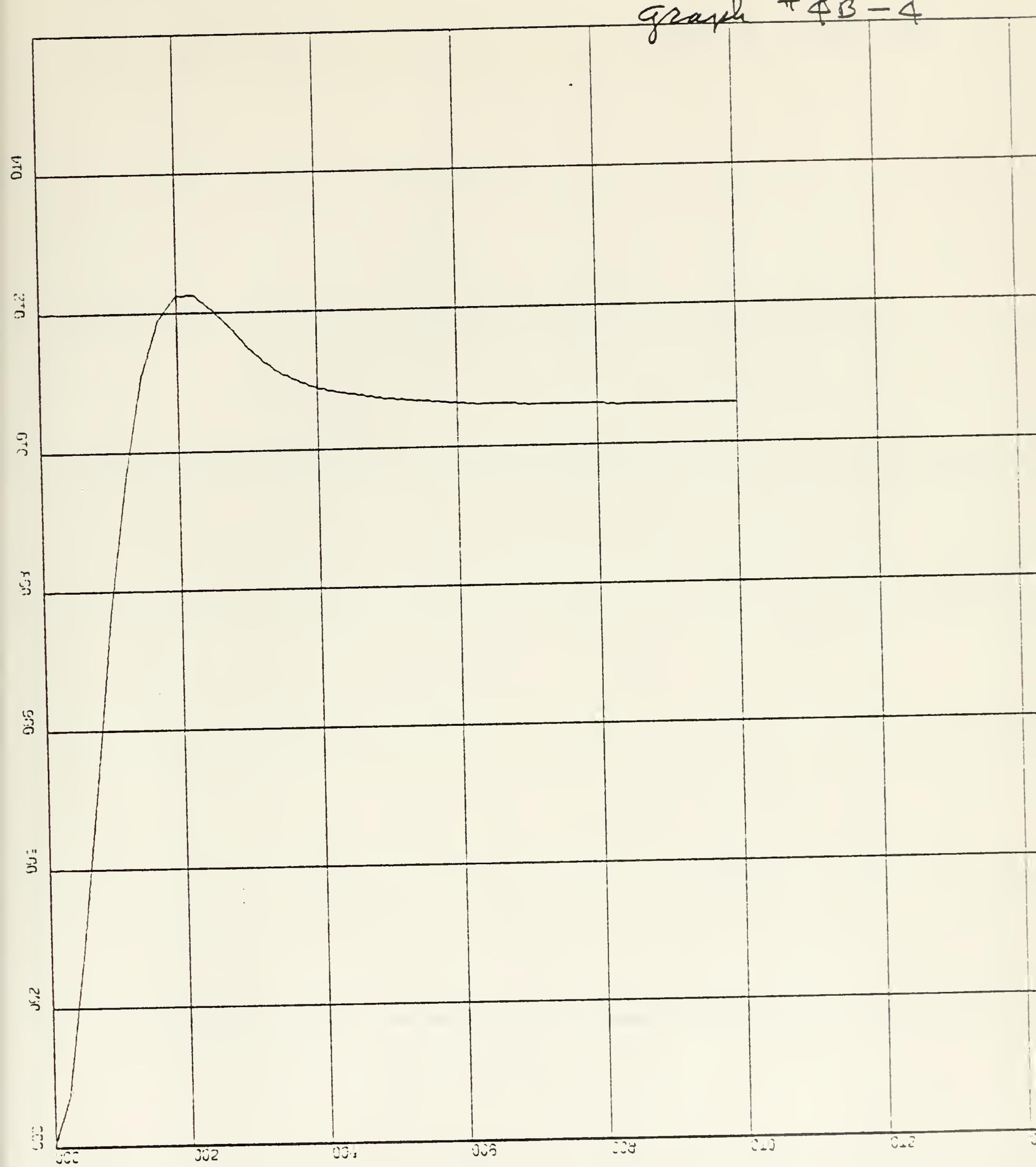


X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=5.00E-05 UNITS INCH.

MODEL-4B

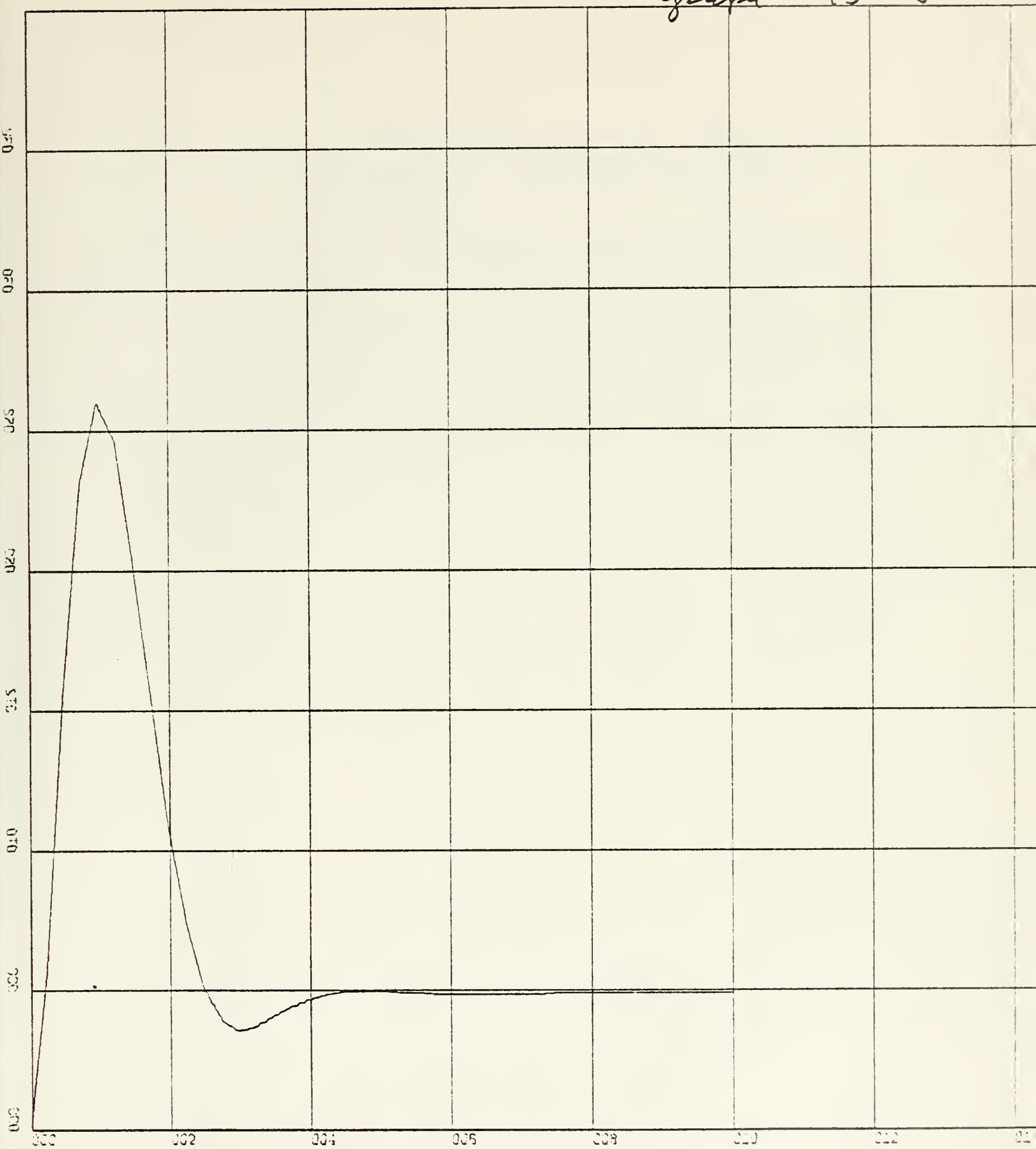
RUN 1 MX-1 VS TIME

Graph #4B-4



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=2.00E-04 UNITS INCH.
MODEL-4B
RUN 1 MY-1 VS TIME

graph #4B-5



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=5.00E-05 UNITS INCH.

MØDEL-4B

RUN 1 M-X1*Y1 VS TIME

LIST OF REFERENCE

(1) David J. Bartholomew, Stochastic Models for Social Processes, John Wiley and Sons.

(2) Merk, J. W, "A Markovian Model for Projection Movements of personnel Through a System"

(3) Gaver, D. P, and Lehoczky, J. P, "Gaussian Approximations to Service Problems."

(4) Ross, N. S, Introduction to Probability Models, Academic press.

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